Equations and Inequalities - Alpha
FAMAT State Convention 2004

The abbreviation NOTA denotes "None Of These Answers."

1. Which is NOT a solution of the inequality $|1-2x|>5$

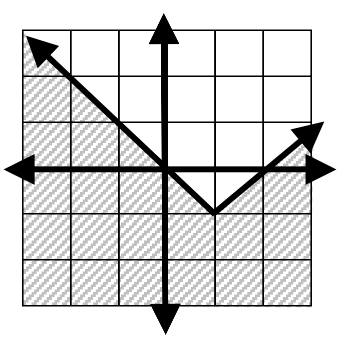
A. -3 B. 3 C. 10
D. 20 E. NOTA

2. Which statement about the equation $\frac{x}{2}-\frac{y}{3}=1$ is true?

A. The graph is a hyperbola.
B. The graph has a domain $x<0$.
C. The y-intercept of the graph is -3.
D. The graph is a parabola.
E. NOTA

3. If $4(5-10x)-2(x+3)=12$ then $42x=$

A. 1 B. 2 C. 3 D. 4 E. NOTA

4. 

Which inequality below has the solution closest to that shown?

A. $y\le|x+1|-1$
B. $y\le|x-1|-1$
C. $y\ge|x-1|-1$
D. $y\ge|x+1|-1$
E. NOTA

5. The equation $x(t)=-16t^2+64t+40$ for $x(t)\ge 0$ gives the distance above ground, in feet, of an object thrown upward, at t seconds. What is the time when the object is highest above ground?

A. t=1 second B. t=2 seconds
C. t=3 seconds D. t=4 seconds
E. NOTA

6. For $i=\sqrt{-1}$, $f(x)=\frac{x-i}{x+i}$. What is the value of $f(1)$?

A. 1 B. 2
C. i D. $-i$ E. NOTA

7. Which equation gives the graph of a parabola with latus rectum of length 10?

A. $y-1=10(x+2)^2$
B. $y-1=-10(x+2)^2$
C. $10(y-1)^2=(x+2)$
D. $(y-1)^2=-10(x+2)$
E. NOTA

8. The equation $12=3^{x-1}$ has real solution k. Give the value of $3k^2$ to the nearest hundredth place.

A. 95.76 B. 46.04
C. 31.92 D. 15.35 E. NOTA

9. Which is the solution set to the inequality $x^2-4x\ge 5$?

A. $[-5,1]$ B. $(-\infty,-1]\cup[5,\infty)$
C. $[-1,5]$ D. $(-\infty,-5]\cup[1,\infty)$
E. NOTA

10. A squirrel climbs a tree with distance above ground (in feet) at time t seconds given by $s(t)=3t+10$ where s is positive. A cat climbs the tree with distance above ground (in feet) at time t seconds given by $c(t)=0.5t+1$. At what time will the cat be exactly 221.5 feet below the squirrel ?

A. 85 sec. B. 87 sec.
C. 92 sec. D. 101 sec. E. NOTA

11. The graph of $f(x)=(x-2)^2(x+3)(x-4)^3$ is tangent to the x-axis at x=

A. 2 B. 3
C. 4 D. -3 E. NOTA

12. If $\frac{x}{y}<\frac{7}{4}$ for integers x and y, then how many possible values of x exist?

A. 6 B. 5 C. 4
D. 0 E. NOTA

13. For all $x<2$ the graph of $f(x)$ has slope $-\frac{2}{3}$, and for all $x>2$, the graph of $f(x)$ has slope $\frac{2}{3}$. If the equation of the graph is $f(x)=a|x-b|+c$ then give the value of b.

A. -6 B. -3 C. 2
D. 3 E. NOTA

14. If $x=\sqrt{40x-\sqrt{40x-\sqrt{40x-\sqrt{\ldots}}}}$ for x a positive integer, then $x=$

A. 42 B. 41
C. 40 D. 39 E. NOTA

15. [x] is defined as the greatest integer value of x less than or equal to x. If $[-\pi]+\left[\sin\frac{\pi}{3}\right]=-k$ then $k=$

A. 5 B. 4 C. 3
D. 2 E. NOTA

16. If $f(x)=x^2\bullet(x+1)^3\bullet(x+2)$ and $f(k)=37800$ then $k+1=$

A. 4 B. 5
C. 6 D. 7 E. NOTA

17. A particle moves along the x-axis with distance from the origin given by $x(t)=\left|-16t^2+48t+2\right|$ at t seconds, for $t\geq 0$. For the time interval [0, 4] seconds, what is the greatest distance that the particle is, from the origin?

A. 1.5 B. 34
C. 38 D. 62 E. NOTA

18. If $-2x<y<2x$ for integers x and y and $x+y=5$ then which is the smallest possible value of x?

A. 1 B. 2
C. 3 D. 4 E. NOTA

19. For positive integers t,y,w,z, it is true that $t+5<y$ and $y>z$ and $w<z-1$. Which statement below **must** be true?

A. $t>z$ B. $z>t$ C. $w<z$
D. $y<z$ E. NOTA

20. For $0 < x < \frac{\pi}{2}$, $\frac{\sin x + 1}{\sin x - 1} + \frac{1}{1 - \sin x} = -1$.

Give the value of $\frac{3x}{4\pi}$.

A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. 1 E. NOTA

21. If $\dfrac{\frac{1}{x+1}}{\frac{1}{2} + \frac{1}{x+1}} = \frac{4}{7}$ then $\frac{1}{x} =$

A. 3 B. 2
C. 1 D. $\frac{1}{2}$ E) NOTA

22. Consider the equation $y = \sum_{x=0}^{k} 3^x$. What is the least value of k for which $y > 9840$?

A. 6 B. 7
C. 8 D. 9 E. NOTA

23. A man makes money in dollars at the end of day d, according to equation $M(d) = 20 + M(d-1)$ for $d > 0$. M(0)=0. That is, at the end of day 6, he makes M(6) dollars, for that day only (not a cumulative total). Tell how much money the man will make on day 30.

A. \$580 B. \$600
C. \$620 D. \$640 E. NOTA

24. If $a + b = 5$ and $ab = 2$ then which is the value of $a^3 + b^3$?

A. 125 B. 117
C. 98 D. 95 E. NOTA

25. If x, y and z are positive integers such that $x < y < z$, then which of the following could be equal to $(y^x)(y^z)$?

A. 1 B. 8 C. 64
D. 81 E. NOTA

26. The graphs of $y = 5$, $y = 1$, $y = 2x + 5$ and $y = 10 - x$ bound a quadrilateral region with two parallel horizontal sides. Find the area of that quadrilateral.

A. 22 B. 26
C. 32 D. 34 E. NOTA

27. The area A of a triangle is given by $A > 10\sin\theta$, for θ an angle of the triangle between the two longest sides. If the triangle has integral length sides, then which three numbers below could be the side lengths of the triangle?

A. 3, 4, 5 B. 1, 1, 1
C. 2, 4, 4 D. 3, 5, 5 E. NOTA

28. The roots of $y = 2x^3 - 3x^2 + 4x + 8$ are r, s and t. What is the value of $rs + st + rt$?

A. $\frac{3}{2}$ B. 2 C. 3 D. 4 E. NOTA

29. Consider the base-five number x= 21_{five} and the base-six number y= 100_{six}. Which is the value of K, for $K = |x - y|$ if K is written in base ten?

A. 81 B. 36
C. 25 D. 12 E. NOTA

30. A body cools after a murder, according to the equation $\ln(T - 80) = kt + C$ where T is the degrees Fahrenheit of the body at t minutes. C and k are constants. If the body was found at 9:00 PM and was 96 ^{o}F, and six minutes later the body was 94 ^{o}F, then tell which person below **could not** have been the murderer. Assume normal body temperature of 98.6 ^{o}F at the time of murder, and all times below are in the time intervals most recent to the murder. Also assume that any murder was done by one person, and no murder for hire occurred.

A. Colonel Mustard who has an alibi from 8:00 PM to 8:30 PM.
B. Dame Ruby who has an alibi from 8:31 PM to 8:40 PM.
C. General Germ who has an alibi from 8:41 PM to 8:50 PM.
D. Madame Mauve who has an alibi from 8:51 PM to 8:59 PM.
E. NOTA

Solutions:

1. Substitute or use $1-2x>5$ *or* $-(1-2x)>5$ which solves to $x<-2$ *or* $x>3$. Choice B is not a solution. **B is the answer**.

2. **C**. The y-intercept is -3.

3. Solve to x=1/21, and so 42x = 2. **Choice B**.

4. **Choice B**: vertex at (1, -1), shaded under.

5. The highest point is at the vertex of the parabolic graph: $t=\frac{-b}{2a}$ = 2. **Choice B**.

6. For $\frac{1-i}{1+i}=\left(\frac{1-i}{1+i}\right)\frac{(1-i)}{(1-i)}=\frac{-2i}{2}=-i$
Choice D.

7. All equations shown have length of LR that is 1/10, except choice D. That choice is correct. **D**.

8. $\ln(12)=\ln(3)^{x-1}$ and $\ln 12=(x-1)\ln 3$.
$\frac{\ln 12}{\ln 3}=x-1$. Add 1 and approximate to get 3.2619. Square and then multiply by 3 to get approximately 31.92, **choice C**.

9. $x^2-4x-5=(x-5)(x+1)$ which gives roots x= 5, x= -1. So putting these numbers on the real number line and checking intervals when $x^2-4x-5\geq 0$ gives interval **choice B**.

10. $(3t+10)-221.5=0.5t+1$ solves to t=85.
Choice A.

11. The squared term will give the root at which the graph is tangent to the x-axis. **Choice A.**

12. x and y can be negative, and 7/4 could have been reduced. There are an infinite number of values. **Choice E**.

13. The vertex must be at x=2, when the slope changes. So b must be 2. **Choice C.**

14. $x=\sqrt{40x-x}$ so $x^2=40x-x$ and $x^2=39x$, and since x is not zero, we have x=39. **Choice D**.

15. $[-3.14...]=-4$ and $\left[\frac{\sqrt{3}}{2}\right]$ = 0. The sum is -4 so k=4. **Choice B**.

16. 37800= $2^3\bullet 3^3\bullet 5^2\bullet 7$ Since the biggest factor is 7 then x+2 must be 7 and x=5. So we have $5^2\bullet 6^3\bullet 7$. **Choice C**.

17. Using the same technique as in #5 gives the vertex is at t=1.5 and it is at position 38. But at t=4, the particle is at – 62, so distance is 62. **D**

18. $-2x<5-x<2x$: $-x<5<3x$. 3x must be positive. 3x=6, x=2. **Choice B**.

19. If you put the variables on the number line then the only clear fact is that y is greater than any of the other and z>w. **Choice C**.

20. Factor out a -1 from the first denominator:
$\frac{-(\sin x+1)}{1-\sin x}+\frac{1}{1-\sin x}=-1$ gives
$\frac{-\sin x}{1-\sin x}=-1;\ \frac{\sin x}{1-\sin x}=1$. sinx=1-sinx. 2sinx=1 and sinx = 1/2 to give x = $\frac{\pi}{6}$ and $\frac{\pi}{6}\bullet\frac{3}{4\pi}=\frac{1}{8}$.
Choice A.

21. Mutliply the numerator and denom. of the large fraction by 2(x+1): $\frac{\frac{1}{x+1}}{\frac{1}{2}+\frac{1}{x+1}}=\frac{2}{(x+1)+2}$

21. continued: So $\frac{2}{x+3}=\frac{4}{7}$ which solves to 0.5. The reciprocal is therefore 2. **Choice B**.

22. The sum of a geometric series is $a_1\left(\frac{1-r^n}{1-r}\right)$

so $3^0\left(\frac{1-3^n}{1-3}\right)>9840$ gives $1-3^n<-19680$ and

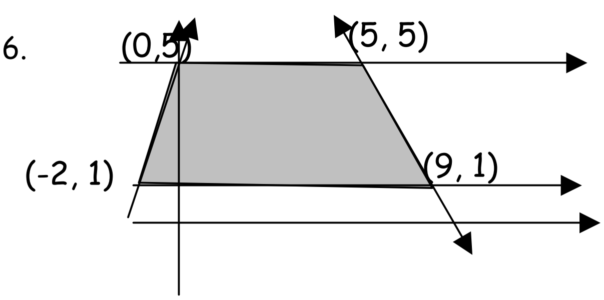
$3^n-1>19680$. Add one, take the ln of both sides, and solve for n to get n> 8.9999075. So the TERM is the 9th term, and k=8. **Choice C**.

23. M(0)=0, M(1)=20, M92)=40, etc and so M(x)=20x. And so M(30)=20(30)=600. Choice B.

24. Square the first equation to get
$a^2+2ab+b^2=25; a^2+b^2=25-4=21$.
$a^3+b^3=(a+b)(a^2-ab+b^2)=$ 5(21-2)=95.
Choice D.

25. In each case: choice A has y=1 which gives no value for x. Choice B gives y=2 and then x must be 1 which gives z=2 which can't be true. Choice C gives y can be either 2 or 4 or 8. The only possibility is that x=1, y=2 and z=5. This may be true. Choice D gives y=3 or 9, and both are impossible when one tries to get x and z. The only possibility is **choice C**.

26.



Getting the intersection points, we have a trapezoid with height 4 and bases 5 and 11. The area is therefore 32, choice C.

27. Using $Area=\frac{1}{2}(side)(side)(\sin(included\ angle))$ we use for sides a and b and included angle theta: $\frac{1}{2}(ab)\sin\theta>10\sin\theta$. Multiply by 2 and divide by a positive number, sinθ, and we get $ab>20$. The only possibility, where the two largest sides multplies to greater than 20 is **choice D**.

28. For ax^3+bx^2+cx+d, the sum of the roots is -b/a. The sum taken two at a time (rs+st+rt) is c/a and the sum taken three at a time (rst) is -d/a. The answer is c/a which is 4/2 which is 2. **Choice B.**

29. 21 (base 5)=11 and 100 (base six)= 36. The difference is 25, choice C.

30. Let t=0 be 9:00. Substitute: ln(96-80)=0+C to give the value of C is ln(16). Then when t=6, we get ln(94-80)=6k+ ln(16) gives the value of k to be ln(14/16) divided by 6. When we use 98.6 and solve for t we get t= -6.766 minutes (approximately). Thus about 6.8 minutes before 9:00, the murder occurred. This is about 8:53-8:54 PM. Madame Mauve had an alibi and thus could not be the murderer. **Choice D**.