Trigonometry

2004 FAMAT State Convention

For all questions, NOTA means None Of The Aforementioned answers is correct.

1. Which of the following angles is coterminal with $9680°$?

   a) $40°$ b) $140°$ c) $220°$ d) $320°$ e) NOTA

2. Let $M = \begin{bmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{bmatrix}$. Calculate MM^T.

   a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ e) NOTA

3. Assume ψ to be a first quadrant angle and λ to be a third quadrant angle. Find $\psi + \lambda$ if

$$\tan(\psi + \lambda) = \frac{\tan(\psi) + 1}{1 - \frac{\sqrt{3}}{3} \cdot \tan(\lambda)}$$

   a) $5\pi/12$ b) $5\pi/4$ c) $17\pi/12$ d) $\pi/4$ e) NOTA

4. Let $f(x) = \sin\left(x + n\frac{\pi}{2}\right)$. Find $f(x)$ for $n = 2004$.

   a) -cos(*x*) b) cos(*x*) c) -sin(*x*) d) sin(*x*) e) NOTA

5. Given $y_1(x) = 10 \cdot \sin(x)$, $y_2(x) = 8 \cdot \cos(x)$, $y_3(x) = y_1(x) + y_2(x)$. Calculate the amplitude of $y_3(t)$ to the nearest tenth.

   a) 12.8 b) 14.7 c) 18.0 d) 19.1 e) NOTA

6. Snell's Law for refracting light from air is follows $\frac{\sin(\Theta_i)}{\sin(\Theta_r)} = k$. At what index of refraction, *k*, is glass if the angle of incidence, Θ_i, is $52°$ and the angle of refraction, Θ_r, is $31.3°$? Round to the nearest tenth.

   a) 1.7 b) 1.5 c) 1.0 d) 0.7 e) NOTA

7. Simplify $f(t) = \frac{1 - \cos(2t)}{2}$.

   a) $-\cos^2(t)$ b) $\cos^2(t)$ c) $\cos(t) - \sin(t)$ d) $\sin(t) - \cos(t)$ e) NOTA

8. Two waves are emitted from a double slit and experience diffraction. They are said to be in phase when $d \cdot \sin(\Theta) = \lambda \cdot m$ (for $m = 0,1,2,\ldots$). For how many ordered pairs (m, Θ) does this equation hold if $d = 2$ and $\lambda = 0.5$? $(0° \le \Theta \le 180°)$

a) 0 b) 4 c) 9 d) 10 e) NOTA

9. Find $A + B$, where A and B are integers satisfying the expression $\sin(3\Theta) = A\sin(\Theta) + B\sin^3(\Theta)$. .

a) -4 b) 2 c) -1 d) 0 e) NOTA

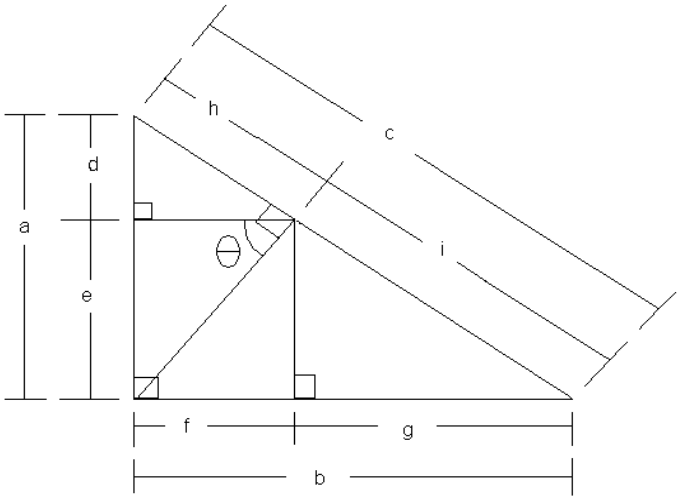
10. Let Θ be a fourth quadrant angle in standard position with $\cos\Theta = \frac{11}{61}$. Evaluate $\sin\Theta$.

a) $\pm\frac{60}{61}$ b) $-\frac{60}{61}$ c) $\frac{60}{61}$ d) 280.4 e) NOTA

11. Evaluate $\left(-2(\cos 216° + i\sin 216°)\right)^5$, where $i = \sqrt{-1}$?

a) 32 b) -32 c) $32i$ d) $-32i$ e) NOTA

12. For the figure below, which of the following represents csc(2θ).



a) $\frac{2fe}{f^2 + e^2}$ b) $\frac{h^2}{2fd}$ c) $\frac{-1}{2ge}$ d) $\frac{2ab}{c^2}$ e) NOTA

13. Determine the angle of rotation needed to eliminate the xy-term from $1x^2 + 2xy - 1y^2 + 8x + 1y = 0$.

a) 60° b) 30° c) 45° d) 22.5° e) NOTA

14. A 240 in. tall wall, standing perpendicular to the ground, casts a shadow. Mary, who is 69 inches tall, stands with her feet at the end of the wall's shadow. She then walks 40 inches towards the wall until her shadow is just within the wall's shadow. How far is Mary standing from the wall, to the nearest inch?

a) 99 in. b) 4 c) 1 d) 0 e) NOTA

15. ΔACM has side lengths a, c, and m opposite vertices A, C, and M, respectively. How many triangles fit the following description: a=21, c=32, $\angle A = 114°$.

a) 3 b) 2 c) 1 d) 0 e) NOTA

16. Given $y(x) = 9\sin(6x + 80) + 23$, calculate the frequency of $y(x)$.

a) $3/\pi$ b) $\pi/3$ c) 6 d) 1/6 e) NOTA

17. Evaluate the following sum for $x = 30°$.

$$1 + \cos(2x) + \cos^2(2x) + \cos^3(2x) + ...$$

a) ∞ b) 4 c) 2 d) 1 e) NOTA

18. Calculate the distance between the polar points $(8, 30°)$ and $(-6, -150°)$.

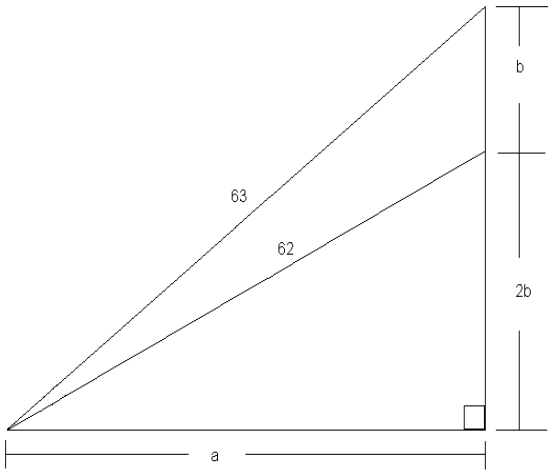
a) $2\sqrt{2}$ b) 2 c) $2\sqrt{3}$ d) 3 e) NOTA

19. Identify the Cartesian equation for the polar graph of $r = \sin\Theta$ for $0 \le \Theta \le 2\pi$.

a) $x^2 + y^2 - x = 0$ b) $x^2 + y^2 - y = 1/4$ c) $x^2 + y^2 - y = 0$

d) $x^2 + y^2 - x = 1/4$ e) NOTA

20. Find b in the right triangle below. (Note: figure not drawn to scale.)



a) $\sqrt{5}$ b) $\sqrt{6}$ c) 10 d) $5\sqrt{5}$ e) NOTA

21. Convert $30°21'18''$ to the nearest 0.0001 radians.

a) 0.9680 b) 0.9582 c) 0.8582 d) 0.1776 e) NOTA

22. Let *E* be an arbitrary even function and *O* be and arbitrary odd function. Each function is defined everywhere over the set of real numbers. Which of the following best describes *E / O*?

a) even b) odd c) neither even nor odd d) both even and odd e) NOTA

23. Which algebraic expression in x corresponds to $\cos(\alpha+\beta)$, where $\alpha=\text{Arccos}(x)$, $\beta=\text{Arccos}(2x)$, where $-0.5 \le x \le 0.5$.

a) $3x$ b) $2x\sqrt{1-x^2} - x\sqrt{1-4x^2}$ c) $x\sqrt{1-4x^2} + 2x\sqrt{1-x^2}$

d) $x\sqrt{1-4x^2} - 2x\sqrt{1-x^2}$ e) NOTA

24. ΔLHS has side lengths l, h, and s opposite vertices L, H, and S, respectively. Given $s = 19$ and $h = 99$ Find the area of ΔLHS for $\sin(\angle L) = 0.5$.

a) 1881 b) 940.5 c) 470.25 d) $470.25\sqrt{2}$ e) NOTA

25. Identify the range of Arctan(x).

a) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ b) $(-\infty,\infty)$ c) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ d) $[-\infty,\infty]$ e) NOTA

26. Evaluate $1+\cot^2(22.5°)$.

a) $4\sqrt{2}+4$ b) $4\sqrt{2}-2$ c) $2\sqrt{2}+2$ d) $2\sqrt{2}-4$ e) NOTA

27. Simplify $\frac{\sin(30°)}{\tan(60°)}\frac{\cot(45°)}{\cos(240°)}\frac{\sec(60°)}{\csc(150°)}\frac{\csc(30°)}{\sin(60°)}\frac{\cos(120°)}{\cot(225°)}\frac{\tan(240°)}{\sec(300°)}$

a) 1 b) -1 c) $\sqrt{2}/2$ d) $-\sqrt{2}/2$ e) NOTA

28. Calculate the angle between vectors $\vec{u}=9i+6j+80k$ and $\vec{v}=-8i-21j-81k$ to the nearest degree.

a) 100° b) 80° c) 170° d) 10° e) NOTA

29. ΔABC has side lengths a, b, and c opposite vertices A, B, and C, respectively. For a = 6, b = 20, c = 25, and $\angle C = \Theta$, evaluate sin ($\angle A$) + sin ($\angle B$) in terms of Θ.

a) $\frac{26}{25}\sin\Theta$ b) $\frac{19}{25}\sin\Theta$ c) $\frac{4}{20}\sin\Theta$ d) $\frac{31}{20}\sin\Theta$ e) NOTA

30. ΔABC has side lengths a, b, and c opposite vertices A, B, and C, respectively. Given $a = 1+\sqrt{3}$, b = 2, $\angle C = \pi/6$, find c.

a) 4 b) 2 c) $\sqrt{10+4\sqrt{3}}$ d) $\sqrt{14+4\sqrt{3}}$ e) NOTA