1) I) $\sum_{n=0}^{\infty}\frac{2}{3^n}$ Converges. Geometric series with r= 1/3. II) $\sum_{n=1}^{\infty}\frac{n^3}{n^2+1}$ Diverges. The integral test fails $\left(\lim_{b\to\infty}\int_1^b \frac{x^3}{x^2+1}dx=\infty\right)$

III) $\sum_{n=1}^{\infty}\frac{1}{3+\sqrt[3]{n}}$ Diverges. Direct comparison test with divergent p-series $\sum_{n=1}^{\infty}\frac{1}{n^{\frac{1}{3}}}$. IV) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$ Converges. Alternating series test.

Limit of 1/n as n→ ∞ is zero and $a_n \geq a_{n+1}$ (1/n ≥ 1/(n+1)). I & IV $\boxed{B}$

2) $S=2\pi\int_0^2 r(x)\sqrt{1+[f'(x)]^2}\,dx$, $r(x)=x$, $f'(x)=2x+2\Rightarrow S=2\pi\int_0^2\left[x\sqrt{4x^2+8x+5}\right]dx$ $\boxed{C}$

3) $r=2\sin(4\theta)$ intersects zero at multiples of $\frac{\pi}{4}$, therefore the limits of integration must be of the same nature. Using the polar area formula, $\frac{1}{2}\int_a^b r^2\,d\theta\Rightarrow\frac{1}{2}\int_0^{\frac{\pi}{4}}(2\sin(4\theta))^2\,d\theta=\frac{\pi}{4}$ $\boxed{B}$

4) Taking successive derivatives of $f(x)$ yields: $f'(x)=\frac{1}{x^2+1}, f''(x)=\frac{-2x}{(x^2+1)^2}, f'''(x)=\frac{(6x^2-2)}{(x^2+1)^3}, f^{(4)}=\frac{(-24x^3+24x)}{(x^2+1)^4}$,

$f^{(5)}=\frac{24(5x^4-10x^2+1)}{(x^2+1)^5}$. $\Rightarrow f(0)=0, f'(0)=1, f''(0)=0, f'''(0)=-2, f^{(4)}=0, f^{(5)}=24$. Forming the Maclaurin series:

$$f(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f^{(3)}(0)}{3!}x^3+\ldots+\frac{f^{(n)}(0)}{n!}x^n+\ldots,\Rightarrow 0+x+0+\frac{-2x^3}{3!}+0+\frac{24x^5}{5!}\ldots\Rightarrow x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}\ldots$$ $\boxed{D}$

5) $\int\frac{5x^2+22x+16}{x(x+2)^2}dx=\int\left(\frac{4}{x}+\frac{1}{x+2}+\frac{4}{(x+2)^2}\right)dx$ (using partial fractions)$\Rightarrow \ln(x+2)-\frac{4}{x+2}+4\ln x+C$ $\boxed{D}$

6) In parametric form, $\frac{d^2y}{dx^2}=\frac{d}{dx}\left[\frac{dy}{dx}\right]=\frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{dx/dt}\Rightarrow\frac{dx}{dt}=6t$, $\frac{dy}{dt}=2\Rightarrow\frac{dy}{dx}=\frac{1}{3t}\Rightarrow\frac{\frac{d}{dt}\left[\frac{1}{3t}\right]}{6t}=-\frac{1}{18t^3}$ $\boxed{D}$

7) By completing the square, $x^2+16y^2+4x-96y+132=0$ becomes $\frac{(x+2)^2}{4^2}+\frac{(y-3)^2}{1^2}=1$. This is the equation of a horizontally aligned ellipse with center (-2,3), major axis 4 and minor axis 1. The area of the revolved surface can be found using the Theorem of Pappus. The area of the ellipse is πab or 4π. The distance that the centroid (center of the ellipse) passes through is $2\pi(6-(-2))=16\pi$. Thus the resulting volume is the product, $(16\pi)(4\pi)=64\pi^2$ $\boxed{A}$

8) $\int\frac{1-\cos 2x}{2}dx=\int\sin^2 x\,dx=\frac{x}{2}-\frac{\sin x\cos x}{2}+C$ $\boxed{E}$

9) $f(x)=(4\sin x)(\arcsin(4x))$. Using the product rule, $f'(x)=(4\cos x)(\arcsin(4x))+\left(\frac{16\sin x}{\sqrt{1-16x^2}}\right)$ $\boxed{A}$

10) Arc length$=\int_a^b\sqrt{1+(f'(x))^2}\,dx$, $f'(x)=\tan x\Rightarrow\int_{-1.5}^{1.5}\sqrt{1+\tan^2 x}\,dx\Rightarrow\int_{-1.5}^{1.5}\sec x\,dx\Rightarrow\left[\ln|\sec x+\tan x|\right]_{-1.5}^{1.5}\approx 6.68$ $\boxed{C}$

11) $\int_{-\pi}^{\pi}\frac{dx}{x^2-8x+20}=\int_{-\pi}^{\pi}\frac{dx}{(x-4)^2+2^2}=\left[\frac{1}{2}\arctan\left(\frac{x-4}{2}\right)\right]_{-\pi}^{\pi}=\frac{\arctan\left(\frac{\pi+4}{2}\right)+\arctan\left(\frac{\pi-4}{2}\right)}{2}$ $\boxed{D}$

12) Polar arc length = $\int_\alpha^\beta\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta\Rightarrow\int_{\frac{\pi}{4}}^{\frac{2\pi}{3}}\sqrt{\sin^2\theta+\cos^2\theta}\,d\theta=\frac{5\pi}{12}$ $\boxed{B}$

13) Surface area $=2\pi\int_a^b r(x)\sqrt{1+[f'(x)]^2}\,dx$. $r(x)=2x^3-6x$, $f(x)=2x^3-6x\Rightarrow$ Surface Area= $2\pi\int_0^{\sqrt{3}}\left[(2x^3-6x)\sqrt{36x^4-72x^2+37}\right]dx$ $\boxed{E}$

14) $\int_0^{\infty}xe^{-x}dx=\left[(-x-1)e^{-x}\right]_0^{\infty}=\lim_{a\to\infty}\left[(-a-1)e^{-a}\right]-\left[(-0-1)e^{-0}\right]=0-1=-1$ $\boxed{A}$

15) $r = 3 - 3\cos\theta$ is a Cardiod, meaning it is symmetric about the polar axis. The only possibly tangent line at 2 points is on the tip of the "heart" portion of the Cardiod. This tangent line must be vertical since the Cardiod is symmetric about the polar axis. To find the vertical tangent, $\frac{dx}{d\theta} = 0 \Rightarrow x = r\cos\theta = (3 - 3\cos\theta)\cos\theta = 3\cos\theta - 3\cos^2\theta . \Rightarrow \frac{dx}{d\theta} = 3\sin\theta(2\cos x - 1)$ $\frac{dx}{d\theta} = 0$ when $\theta = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$.

The two endpoints that are linear and both tangent to the graph are $\left(\frac{3}{2}, \frac{\pi}{3}\right)$ & $\left(\frac{3}{2}, \frac{5\pi}{3}\right)$ $\boxed{D}$

16) $f(x) = 2\arcsin(2x) + 3x\sqrt{1-2x^2} \Rightarrow$ $f'(x) = \frac{4}{\sqrt{1-4x^2}} + 3\sqrt{1-2x^2} - \frac{6x^2}{\sqrt{1-2x^2}}$ (using the product rule) $\boxed{B}$

17) $\int \sin^4\phi\cos^3\phi d\phi = \int \sin^4\phi(\cos^2\phi)\cos\phi d\phi = \int\left[\sin^4\phi - \sin^6\phi\right]\cos\phi d\phi = \frac{\sin^5\phi}{5} - \frac{\sin^7\phi}{7} + C$ $\boxed{B}$

18) $u = \arctan x$, $du = \frac{dx}{1+x^2}$, $dv = 1dx$, $v = x \Rightarrow$ $x\arctan x - \int \frac{x}{1+x^2}dx =$ $x\arctan x - \frac{\ln(x^2+1)}{2} + C \cdot \int \arcsin x dx \Rightarrow u = \arcsin x$, $du = \frac{dx}{\sqrt{1-x^2}}$,

$dv = 1dx$, $v = x \Rightarrow$ $x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}dx =$ $x\arcsin x - \sqrt{1-x^2} + C \Rightarrow\Rightarrow x\arctan x + x\arcsin x + \sqrt{1-x^2} - \frac{\ln(x^2+1)}{2} + C$ $\boxed{C}$

19) $\lim\limits_{x\to\infty}\frac{x^3}{2e^{2x}} \Rightarrow \frac{\infty}{\infty}$, use L'Hopital's Rule$\Rightarrow$ $\lim\limits_{x\to\infty}\frac{3x^2}{4e^{2x}}$ $\frac{\infty}{\infty}$, use L'Hopital's Rule$\Rightarrow$ $\lim\limits_{x\to\infty}\frac{6x}{8e^{2x}}$ use L'Hopital's Rule$\Rightarrow$ $\lim\limits_{x\to\infty}\frac{6}{16e^{2x}} = 0$ $\boxed{A}$

20) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \ldots + \frac{(-1)^n x^{2n}}{2n+1} + \ldots$ is the Maclaurin Series for $y = \cos x$. $\boxed{E}$

21) p-series test for convergence is proven by using the Integral Test. A p-series converges if $p>1$ $\boxed{B}$

22) Area = $\int_2^6\left[(6x - x^2) - (12 - 2x)\right]dx = \frac{32}{3}$, $\bar{x} = \frac{3}{32}\int_2^6\left[x\left((6x - x^2) - (12 - 2x)\right)\right]dx = 4$ $\boxed{D}$

23) Area = $\int_2^6\left[(6x - x^2) - (12 - 2x)\right]dx = \frac{32}{3}$, $\bar{y} = \frac{3}{32}\int_2^6\left[\left(\frac{(6x - x^2) + (12 - 2x)}{2}\right)\left((6x - x^2) - (12 - 2x)\right)\right]dx = \frac{28}{5}$ $\boxed{A}$

24) $\frac{dx}{dt} = 3t^2$, $\frac{dy}{dt} = \frac{1}{2\sqrt{t}} \Rightarrow \frac{dy}{dx} = \frac{1}{6t^{\frac{5}{2}}}$. $(2,1)$ corresponds to $t = 1$. $\frac{dy}{dx}$ at $t = 1$ equals $\frac{1}{6}$. $\frac{d^2y}{dx^2} = \frac{-5}{36t^{\frac{11}{2}}}$ at $t = 1$ equals $-\frac{5}{36}$

thus at $(2,1)$ the graph is concave down. $\boxed{E}$

25) Volume = $2\pi\int_0^3\left(x^{\frac{2}{3}}(3 - x)\right)dx = \frac{81\pi(\sqrt[3]{9})}{20}$ $\boxed{C}$

26) $4 + \frac{8}{3} + \frac{16}{9} + \frac{32}{27} + \ldots = 4\left(\frac{2}{3}\right)^0 + 4\left(\frac{2}{3}\right)^1 + 4\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right)^3 + \ldots \Rightarrow$ Sum=$\frac{a}{1-r}$, where a=4, and r=2/3. S= 12 $\boxed{B}$

27) Area of equilateral triangle= $\frac{s^2\sqrt{3}}{4} \Rightarrow$ Volume=$\int_0^4\left(\frac{(x^3)^2\sqrt{3}}{4}\right)dx = \frac{4096\sqrt{3}}{7}$ $\boxed{C}$

28) $\int\frac{2x^2 - 2x - 3}{x^2 - 1}dx = \int\left(-\frac{1}{2x+2} - \frac{3}{2x-2} + 2\right)dx = -\frac{\ln(x+1)}{2} - \frac{3\ln(x-1)}{2} + 2x + C$ $\boxed{A}$

29) $f(x) = 2\cosh(2x) \Rightarrow \frac{dy}{dx} = 4\sinh(2x)$ $\boxed{B}$

30) Solve for the upper bound: $x^2 = -2x + 4 \Rightarrow x = \sqrt{5} - 1$. $M_y = \rho\int_0^{\sqrt{5}-1}\left[x\left(-2x + 4 - (x^2)\right)\right]dx = \frac{\rho(26 - 10\sqrt{5})}{3}$ $\boxed{D}$