BC Calculus
FAMAT State Convention 2004

For all questions, E. NOTA means none of the above answers is correct.

1) Which of the following series converge?

i) $\sum_{n=0}^{\infty}\frac{2}{3^n}$ ii) $\sum_{n=1}^{\infty}\frac{n^3}{n^2+1}$ iii) $\sum_{n=1}^{\infty}\frac{1}{3+\sqrt[3]{n}}$ iv) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$

A) i, ii B) i, iv C) ii, iv D) iii, iv E) NOTA

2) Which of the following represents the surface area of revolution of the graph $f(x)=x^2+2x$ about the y-axis on the interval $[0,2]$?

A) $2\pi\int_0^2\left[(x^2+2x)\sqrt{x^2+2x+1}\right]dx$ B) $2\pi\int_0^2\left[x\sqrt{x^4+4x^3+4x^2+1}\right]dx$

C) $2\pi\int_0^2\left[x\sqrt{4x^2+8x+5}\right]dx$ D) $2\pi\int_0^2\sqrt{2x+3}dx$ E) NOTA

3) What is the area of one of the petals of the polar graph $r=2\sin(4\theta)$?

A) $\frac{\pi}{8}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{2}$ E) NOTA

4) What is the Maclaurin series for the function $f(x)=\arctan(x)$?

A) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\frac{x^9}{9!}\ldots$ B) $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}\ldots$

C) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}\ldots$ D) $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}\ldots$ E) NOTA

5) Evaluate $\int\frac{5x^2+22x+16}{x(x+2)^2}dx$

A) $\frac{4}{x}+\frac{1}{x+2}+\frac{4}{(x+2)^2}+C$ B) $\frac{4}{x}+\frac{4}{x+2}+\frac{1}{(x+2)^2}+C$

C) $4\ln(x+2)-\frac{1}{x+2}+\ln x+C$ D) $\ln(x+2)-\frac{4}{x+2}+4\ln x+C$ E) NOTA

6) Find $\frac{d^2y}{dx^2}$ for the curve given $x=3t^2$ and $y=2t+2$.

A) $-\frac{1}{3t^2}$ B) $\frac{1}{6t}$ C) $\frac{1}{18t^2}$ D) $-\frac{1}{18t^3}$ E) NOTA

7) Find the volume of revolution formed by revolving the ellipse $x^2+16y^2+4x-96y+132=0$ about the line $x=6$.

A) $64\pi^2$ B) $48\pi^2$ C) $32\pi^2$ D) $16\pi^2$ E) NOTA

8) Evaluate $\int \frac{1-\cos 2x}{2} dx$

A) $-\cos^2 x + C$ B) $\frac{\sin(2x)+2x}{4} + C$ C) $\frac{\sin(2x)-2x}{4} + C$ D) $-\sin^2 x + C$ E) NOTA

9) Find $\frac{dy}{dx}$ where $f(x) = (4\sin x)(\arcsin(4x))$.

A) $(4\cos x)(\arcsin(4x)) + \left(\frac{16\sin x}{\sqrt{1-16x^2}}\right)$ B) $(4\cos x)(\arcsin(4x)) + \left(\frac{4\cos x}{\sqrt{1-16x^2}}\right)$

C) $(4\sin x)(\arccos(4x)) + \left(\frac{16\sin x}{\sqrt{1-16x^2}}\right)$ D) $(4\cos x)(\arcsin(4x)) + \left(\frac{4\sin x}{\sqrt{1-4x^2}}\right)$ E) NOTA

10) What is the arc length, to the nearest hundredth, of the graph of $y = -\ln(\cos x)$ on the interval $\left(-\frac{3}{2}, \frac{3}{2}\right)$?

A) 20.99 B) 28.20 C) 6.68 D) 5.31 E) NOTA

11) Evaluate $\int_{-\pi}^{\pi} \frac{dx}{x^2 - 8x + 20}$

A) $\frac{1}{2}\left(\ln\left(\frac{(\pi+4)}{2}\right) + \ln\left(\frac{(\pi-4)}{2}\right)\right)$ B) $\frac{\ln(\pi+4) - \ln(\pi-4)}{2} + \frac{\arctan(\pi+4) - \arctan(\pi+4)}{2}$

C) $\frac{1}{4}(\arctan(\pi+2) + \arctan(\pi-2))$ D) $\frac{1}{2}\left(\arctan\left(\frac{\pi+4}{2}\right) + \arctan\left(\frac{\pi-4}{2}\right)\right)$ E) NOTA

12) What is the arc length of the polar function $r = \sin\theta$ on the interval from $\theta = \frac{\pi}{4}$ to $\theta = \frac{2\pi}{3}$.

A) $\frac{\sqrt{3}-\sqrt{2}}{2}$ B) $\frac{5\pi}{12}$ C) $\frac{\sqrt{2}-1}{2}$ D) $\frac{7\pi}{12}$ E) NOTA

13) Which of the following integrals represents the surface area of revolution of the graph of $y = 2x^3 - 6x$ about the x-axis on the closed interval from $x = 0$ to $x = \sqrt{3}$?

A) $2\pi \int_0^{\sqrt{3}} \left[x\sqrt{36x^4 - 72x^2 + 37}\right] dx$ B) $\pi \int_0^{\sqrt{3}} \left[(2x^3 - 6x)\sqrt{36x^4 - 72x^2 + 37}\right] dx$

C) $2\pi \int_0^{\sqrt{3}} \left[12x(x^2 - 3)(x^2 - 1)\right] dx$ D) $\pi \int_0^{\sqrt{3}} \left[12x(x^2 - 3)(x^2 - 1)\right] dx$ E) NOTA

14) Which of the following improper integrals converges to a defined value?

A) $\int_0^{\infty} xe^{-x} dx$ B) $\int_0^{\infty} \frac{1}{x^3} dx$ C) $\int_1^{\infty} \frac{1}{x} dx$ D) $\int_0^{\infty} \cos(\pi x) dx$ E) NOTA

15) A tangent line can be drawn on the polar graph $r = 3 - 3\cos\theta$ such that the line is tangent to exactly two distinct points on the graph. What are the polar coordinates of these two points?

A) $\left(\frac{9}{2}, \frac{2\pi}{3}\right)$ & $\left(\frac{9}{2}, \frac{4\pi}{3}\right)$ B) $\left(1, 3 - \frac{3\sqrt{3}}{2}\right)$ & $\left(1, 3 + \frac{3\sqrt{3}}{2}\right)$

C) $(6, \pi)$ & $(0, 0)$ D) $\left(\frac{3}{2}, \frac{\pi}{3}\right)$ & $\left(\frac{3}{2}, \frac{5\pi}{3}\right)$ E) NOTA

16) Find $f'(x)$ of the function $f(x) = 2\arcsin(2x) + 3x\sqrt{1-2x^2}$.

A) $\sqrt{1-2x^2} - \frac{3x^2}{\sqrt{1-2x^2}} + \frac{4}{\sqrt{1-4x^2}}$ B) $\frac{4}{\sqrt{1-4x^2}} + 3\sqrt{1-2x^2} - \frac{6x^2}{\sqrt{1-2x^2}}$

C) $\frac{4}{\sqrt{1-2x^2}} + 3\sqrt{1-2x^2} - \frac{3x^2}{\sqrt{1-2x^2}}$ D) $\frac{2}{\sqrt{1-4x^2}} + \sqrt{1-2x^2} - \frac{3x^2}{\sqrt{1-2x^2}}$ E) NOTA

17) Evaluate $\int (\sin^4\phi \cos^3\phi) d\phi$

A) $\frac{\sin^5\phi}{5} + \frac{\cos^4\phi}{4} + C$ B) $\frac{\sin^5\phi}{5} - \frac{\sin^7\phi}{7} + C$ C) $\frac{\cos^5\phi}{5} - \frac{\cos^7\phi}{7} + C$ D) $\frac{\cos^5\phi}{5} + \frac{\cos^6\phi}{6} + C$ E) NOTA

18) Evaluate $\int (\arcsin x + \arctan x) dx$

A) $\frac{1}{2}(\arcsin^2 x + \arctan^2 x) + C$ B) $x\arctan x + x\arcsin x + \frac{-2x}{\sqrt{1-x^2}} - \frac{\ln(x^2+1)}{2} + C$

C) $x\arctan x + x\arcsin x + \sqrt{1-x^2} - \frac{\ln(x^2+1)}{2} + C$ D) $\arctan x + \arcsin x + \sqrt{1-x^2} - \frac{\ln(x^2+1)}{2} + C$ E) NOTA

19) Evaluate $\lim_{x\to\infty} \frac{x^3}{2e^{2x}}$

A) 0 B) 1 C) ∞ D) Does Not Exist E) NOTA

20) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \ldots + \frac{(-1)^n x^{2n}}{2n+1} + \ldots$ is the Maclaurin Series for which of the following?

A) $y = \sin x$ B) $y = \ln x$ C) $y = \tan x$ D) $y = e^x$ E) NOTA

21) Make the following a true statement: The *p*-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, with n as a positive integer, _____.

A) converges if $p < 0$ B) converges if $p > 1$ C) converges if $0 < p < 1$ D) converges if $p < 1$ E) NOTA

22) What is the x-coordinate of the centroid of the region bounded by the functions $f(x) = 6x - x^2$ and $g(x) = 12 - 2x$?

A) $\frac{21}{5}$ B) 3 C) $\frac{15}{4}$ D) 4 E) NOTA

23) What is the y-coordinate of the centroid of the region bounded by the functions $f(x)=6x-x^2$ and $g(x)=12-2x$?

A) $\frac{28}{5}$ B) 4 C) $\frac{56}{5}$ D) $\frac{71}{10}$ E) NOTA

24) Find the slope and the concavity for the curve given by $x=t^3+1$ and $y=\sqrt{t}$ at the point $(2,1)$.

A) $\frac{3}{2}$; concave up B) $\frac{2}{3}$; concave down C) $\frac{1}{6}$; concave up D) $\frac{3}{2}$; concave down E) NOTA

25) What is the volume of the figure formed by revolving the region bounded by $y=x^{\frac{2}{3}}$, $y=0$, and $x=3$ about the line $x=3$?

A) $\frac{81\pi\left(\sqrt[3]{9}\right)}{40}$ B) $\frac{54\pi\left(\sqrt[3]{3}\right)}{7}$ C) $\frac{81\pi\left(\sqrt[3]{9}\right)}{20}$ D) $\frac{235\pi\left(\sqrt[3]{3}\right)}{35}$ E) NOTA

26) What is the sum of the infinite geometric series $4+\frac{8}{3}+\frac{16}{9}+\frac{32}{27}+\ldots$?

A) 11 B) 12 C) 13 D) $\frac{49}{4}$ E) NOTA

27) What is the volume of the three dimensional figure formed by taking equilateral triangle cross sections perpendicular to the x-axis along the graph of $y=x^3$ on the interval $(0,4)$?

A) $16\sqrt{3}$ B) $\frac{512\sqrt{3}}{3}$ C) $\frac{4096\sqrt{3}}{7}$ D) $\frac{16\sqrt{3}}{3}$ E) NOTA

28) Evaluate $\int\frac{2x^2-2x-3}{x^2-1}dx$

A) $-\frac{\ln(x+1)}{2}-\frac{3\ln(x-1)}{2}+2x+C$ B) $\frac{\ln(x+1)}{2}-\frac{\ln(x-1)}{2}+2+C$

C) $\frac{1}{2}\arctan\left(\frac{2x^2-2}{x-1}\right)+C$ D) $\ln(x+1)-3\ln(x-1)+2x+C$ E) NOTA

29) $f(x)=2\cosh(2x)$. Find $\frac{dy}{dx}$.

A) $-\sinh(2x)$ B) $4\sinh(2x)$ C) $-2\sinh(2x)$ D) $-4\sinh(2x)$ E) NOTA

30) What is the moment about the y-axis of the region bounded by $f(x)=-2x+4$, $g(x)=x^2$, and the y-axis with uniform density ρ?

A) $\rho\left(\frac{10\sqrt{5}-14}{3}\right)$ B) $\rho\left(\frac{40\sqrt{5}}{3}-\frac{376}{15}\right)$ C) $\rho\left(\frac{80\sqrt{5}}{3}-\frac{752}{15}\right)$ D) $\rho\left(\frac{26-10\sqrt{5}}{3}\right)$ E) NOTA