Calculus Applications
Mu Alpha Theta State Convention 2004

Note: The answer choice E (NOTA) stands for "none of the above"

1) A light is at the top of a 20 foot pole. Kemp (who is 8 feet tall) walks away from the pole at the speed of 8 ft/s. At what rate is the length of his shadow increasing?

(A) $\frac{8}{3} ft/s$ (B) $\frac{16}{3} ft/s$ (C) $-\frac{8}{3} ft/s$ (D) $16 ft/s$ (E) NOTA

2) The centroid of the plane region bounded by $y = x^4$ and $y = 16$ is

(A) $\left(0, \frac{45}{4}\right)$ (B) $(0,8)$ (C) $\left(0, \frac{80}{9}\right)$ (D) $\left(0, \frac{60}{7}\right)$ (E) NOTA

3) What is the length of the curve $r = 8(\sin\theta - 1)$?

(A) 64 (B) 32 (C) 16 (D) 8 (E) NOTA

4) If a particle moves according to $r(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$, what is its speed at $t = 2$?

(A) $\sqrt{161}$ (B) $\sqrt{137}$ (C) $\sqrt{87}$ (D) $2\sqrt{21}$ (E) NOTA

5) A ball is thrown directly upwards at an initial speed of $2000 ft/s$. Ignoring any air resistance, what is the maximum height achieved by this ball? (Assume g = $-32 ft/s^2$)

(A) $20{,}000 ft$ (B) $62{,}500 ft$ (C) $64{,}000 ft$ (D) $81{,}000 ft$ (E) NOTA

6) What is the volume of the torus created by rotating the circle $x^2 + y^2 = 4$ around the line $x = 5$? (Round your answer to the nearest hundredth)

(A) 114.15 (B) 112.24 (C) 135.79 (D) 163.88 (E) NOTA

7) Find the area of the region between the curves $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$.

(A) 12 (B) 18 (C) 22 (D) 24 (E) NOTA

8) What is the value of $\int_0^{\pi/4} x \tan x\,dx$ using the trapezoidal rule with n = 4? (Approximate your answer to the nearest thousandth)

(A) 0.182 (B) 0.186 (C) 0.194 (D) 0.198 (E) NOTA

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9) Alex has a gambling problem. Five months after he started playing, his life savings had dropped from $200,000 to $80,000 due to losses at the blackjack and poker tables. What is funny about these losses (though loss of money should not be considered funny) is that over the past five months, his life savings have followed an exponential decline. Assuming his money continues to plummet at the same exponential rate, how much money will Alex have after another 3 months? (Round to the nearest hundred dollars)

(A) $46,200 (B) $46,300 (C) $46,000 (D) $46,100 (E) NOTA

10) Mauro has been inflating a balloon in the shape of a spherical cow since morning. The volume of the balloon(in m^3) t minutes after 12:00 PM is given by $V(t) = \frac{4\pi}{3}(1+3t)^3$. What is the rate of change with respect to t at 12:24 PM of the surface area of the balloon? (Round to the nearest hundredth)

(A) $5504.06\frac{m^2}{\min}$ (B) $5504.07\frac{m^2}{\min}$ (C) $5405.06\frac{m^2}{\min}$ (D) $5405.07\frac{m^2}{\min}$ (E) NOTA

11) Using two iterations of Newton's method, what is a zero of the function $f(x) = 2x^3 - 4x + 3$, given $x_0 = 2$? (Approximate to the nearest hundredth).

(A) 1.45 (B) 1.46 (C) 1.07 (D) 1.09 (E) NOTA

12) Evaluate the integral $\int \frac{1}{-\sqrt{6v - v^2}}\,dv.$

(A) $\arccos(\frac{x-3}{3}) + C$ (B) $\arcsin(\frac{x-6}{6}) + C$

(C) $\arcsin(\frac{x-3}{3}) + C$ (D) $\frac{1}{2}\arccos(\frac{x-3}{3}) + C$ (E) NOTA

13) At what point does the line tangent to $f(x) = 2x^3 - 4x + 3$ at $x = 3$ intersect with the perpendicular to the tangent of $g(x) = 2x^2 - \frac{3}{8}$ at $x = \frac{1}{4}$?

(A) $\left(\frac{1}{12}, \frac{22}{3}\right)$ (B) $\left(\frac{-1}{12}, \frac{11}{6}\right)$ (C) $\left(\frac{-1}{12}, \frac{13}{6}\right)$ (D) $\left(\frac{1}{6}, \frac{11}{12}\right)$ (E) NOTA

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14) What is the volume of the solid of revolution formed by the area bounded by $f(x) = x^3 - x^2$ and $g(x) = -(x-2)^2 + 4$ around the y-axis?

(A) $\frac{128\pi}{15}$ (B) $\frac{256\pi}{15}$ (C) $\frac{128\pi}{45}$ (D) $\frac{256\pi}{45}$ (E) NOTA

15) A quantity of gas with an initial volume of 1 ft^3 and pressure of 800 $\frac{lbs}{ft^2}$ expands to a volume of 5 ft^3. Find the work done by this gas in $ft \cdot lbs$. (Assume that pressure is inversely proportional to the volume; round to the nearest hundredth)

(A) 815.14 $ft \cdot lbs$ (B) 1245.45 $ft \cdot lbs$ (C) 1267.21 $ft \cdot lbs$ (D) 1287.55 $ft \cdot lbs$ (E) NOTA

16) Evaluate the derivative of $g(x) = \tan(x)\cot(x)$ at (3,1).

(A) π (B) $\frac{\pi}{2}$ (C) 1 (D) 0 (E) NOTA

17) Find $h'(4)$ if $h(x) = (g(x))^3 (f(x))^2$ where $g(4) = 5, g'(4) = \frac{1}{4}$, $f(4) = 8, f'(4) = 2$

(A) 12000 (B) 11200 (C) 10400 (D) 9060 (E) NOTA

18) A probability density function is to have the form $p(x) = \frac{h^2}{x^4}$ on the interval $[1, \infty)$. What is the value of h?

(A) π (B) 16 (C) 2 (D) $\sqrt{3}$ (E) NOTA

19) A ship is following a counterclockwise course along the curve $r = 6\cos\theta$. A radar station at the origin first spots the ship at $\left(3\sqrt{2}, \frac{\pi}{4}\right)$. The station then notes that, 10 minutes later, the ship is at $\left(3, \frac{\pi}{3}\right)$. Assuming constant speed, how long will it take the ship to reach the radar station from $\left(3, \frac{\pi}{3}\right)$?

(A) 5 min (B) 10 min (C) 20 min (D) 25 min (E) NOTA

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20) How many asymptotes does the following function have?

$$A(x) = \frac{2x^3 + 10x^2 - 48x}{x^3 - 8x^2 - 9x + 72}$$

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

21) Integrate: $\int \frac{1}{x^{1/2} + 2} dx.$

(A) $2\sqrt{x} - 4\ln\left|\sqrt{x} + 2\right| + C$ (B) $\sqrt{x} - 2\ln\left|\sqrt{x} + 2\right| + C$

(C) $\frac{\sqrt{x^{1/2} + 2}}{x^{1/2}} + C$ (D) $\frac{2\sqrt{x^{1/2} + 2}}{x^{1/2}} + C$ (E) NOTA

22) Find 2 non-negative numbers (x, y) where the product xy^2 is maximized, given that $x + y = 9$.

(A) (9,0) (B) (0,9) (C) (3,6) (D) (6,3) (E) NOTA

23) If Alex flies a kite which is 45 ft. above him with the wind having a 10 ft/s horizontal velocity, at what rate is the string tied to the kite being pulled out when its length is 51 ft. (answer in ft/min to the nearest hundredth)?

(A) $4.71\frac{ft}{\min}$ (B) $263.12\frac{ft}{\min}$ (C) $282.35\frac{ft}{\min}$ (D) $6.82\frac{ft}{\min}$ (E) NOTA

24) How many points of inflection does $f(x) = x^3 e^{2x}$ have?

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

25) Evaluate: $\int_0^{\pi/4} \frac{\cos 2x}{|\cos x + \sin x|} dx$

(A) $\sqrt{2}$ (B) 1 (C) $\sqrt{2} - 1$ (D) 1 - $\sqrt{2}$ (E) NOTA

26) Compute the volume of the solid created by rotating the function $f(x) = x^3 - 2x^2 - 1$ around the y axis, bounded by $3 \le x \le 4$ and the x-axis.
(Round to the nearest integer)

(A) 2573 (B) 410 (C) 2574 (D) 411 (E) NOTA

27) What is the curvature at the point $\left(1, \frac{3\sqrt{3}}{2}\right)$ of the curve given by the parametric equations $x = 2\sin t$ and $y = 3\cos t$? (Estimate to the nearest thousandth)

(A) 1.309 (B) 16.039 (C) 0.062 (D) 0.764 (E) NOTA

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28) Hahvahd University is known for its amazingly large and rapidly rising endowment. The rate of change of this endowment is known to be proportional to the current endowment minus 15 (in Billions). At the end of 2001 (t=0), Hahvahd's endowment was approximately 18.3 Billion. At the end of 2003 (t=2), Hahvahd's endowment was approximately 19.3 Billion. What is the estimated endowment for Hahvahd in 2007 (t=6) in Billions? (Round to the nearest tenth)

(A) 20.3 (B) 20.6 (C) 21.3 (D) 22.3 (E) NOTA

29) Use Simpson's rule where n = 3 to approximate $\int_0^6 x^2 e^{x/2} dx$. (Round to the nearest thousandth)

(A) 558.399 (B) 884.420 (C) 965.441 (D) 6178.516 (E) NOTA

30) Kevin has been thinking a lot about the future recently. He decides to bury a time capsule in his backyard containing some of his most prized possessions (including his autobiography), in hopes that future generations will be able to learn much from him and his beliefs. Kevin also happens to be quite the keen scientist. Thus, within the capsule, he writes "180g" under the lid, to correspond to the amount of Carbon-14 within the capsule. Many years later, Kevin's great^{10} grandson Antony finds this capsule in the backyard. It seems that the love of science has been maintained throughout the generations and Antony finds the current level of Carbon-14 to be 172g. How many years ago does Antony calculate the capsule to have been buried? (Round to the nearest year) NOTE: The half-life of Carbon-14 is about 5730 years.

(A) 375 years (B) 376 years (C) 455 years (D) 456 years (E) NOTA