Question #1
Calculus Bowl 2004

The graph of function \( f \) is shown in the figure such that
\[ f(1) = 3, f(3) = 1, f(5) = 4, f(7) = 2 \text{ and } f(9) = 5. \]
Three approximations for \( \int_{1}^{9} f(x) \, dx \) are obtained using \( n = 4 \) where \( n \) is the number of equal subdivisions of \([1, 9]\).

L = the left endpoint Riemann sum approximation
R = the right endpoint Riemann sum approximation
T = the trapezoid rule approximation

Find \( L + R + T \).

Note: The scale on both axes is 1.

Question #2
Calculus Bowl 2004

\( f(x) = \arctan(x) \) and \( g(x) = 4 - x^2 \)
Regions R and S are as follows: (Also see diagram)
R = the area of the region bounded by the y-axis and \( f \) and \( g \) in quadrant I
S = the area of the region bounded by the graphs of \( f \) and \( g \) above and below by the x-axis in quadrant I.

Find \( R - S \) to the nearest thousandth.

Question #3
Calculus Bowl 2004

If \( f(x) = ax^2 + bx + c \) has an x-intercept of 3, a y-intercept of 2 and a tangent line with slope 2 at the x-intercept, find \( a + b + c \).

Question #4
Calculus Bowl 2004

A particle is moving along the curve \( \frac{x^2 f'(x)}{2 - f^2(x)} = -4 \). If the x-coordinate is increasing at a constant rate of 2 units per second, find the rate at which the y-coordinate is changing in units per second when the particle is at \((-1, 2)\).
**Question #5**  
*Calculus Bowl 2004*

The velocity of an object moving on a line is given by \( v(t) = \ln(t+2) + 2\sin(2t) - 0.5 \) on \([0, 2]\). The object is located at 2 on the number line when \( t = 0 \). Find the sum of the values of the statements listed that are correct to the nearest thousandth. Values of the statements are listed in parenthesis to the left.

(-3) Speed is increasing at \( t = 1.9 \).
(5) Total distance traveled is 2.955
(-8) The object ends up at 4.813 on the number line.
(7) Acceleration is increasing at \( t = 1.9 \).
(4) The object changes direction once.
(-7) The average velocity is 1.406.

**Question #6**  
*Calculus Bowl 2004*

If \( f(x) = e^{\sin(x)} \), find \( A, B, C, \) and \( D \) to the nearest thousandth.

\( A = \) the linear approximation of \( f(.1) \) using a tangent line at \( x = 0 \).
\( B = \) the area of the region bounded by the axes, \( f(x) \) and \( x = 1 \).
\( C = \) the volume of the solid formed when revolving the region described in \( B \) about the \( x \)-axis.
\( D = \) the least value of \( c \) guaranteed by the Mean Value Theorem for Derivatives on the interval \([0, 1]\).

Find \( A + B + C + D \).

**Question #7**  
*Calculus Bowl 2004*

\( A = \) the rate of change in cm\(^2\) / min of the area of an equilateral triangle when the side is 6 cm in length if the perimeter is increasing at a constant rate of 9 cm / min.

\( B = \) the \( x \)-coordinate of the point on the graph of \( y = 3\ln(x+2) \) that is closest to the origin.

Find \( A + B \) and round the answer to the nearest thousandth.

**Question #8**  
*Calculus Bowl 2004*

Let \( A = \) the exact distance between the critical points on the graph of \( y = x^3 - 2x^2 + x - 1 \).

Let \( B = \) the \( x \)-coordinate of the inflection point on the graph of \( y = x^3 - 2x^2 + x - 1 \).

Find \( \frac{A}{B} \).
Question #9
Calculus Bowl 2004
Let \( f(x) = 2x^2 - 3 + g(x) \), \( f''(x) = 8x - 12 \), and \( g(0) = 1 \). Find \( g(2) \).

Question #10
Calculus Bowl 2004
Find the volume of the solid formed when the region enclosed by \( y = e^{-x}, y = \ln(x + 1) \), and the \( y \)-axis is rotated about \( y = -2 \). Round to the nearest thousandth.

Question #11
Calculus Bowl 2004
The graph shown is made up of a semicircle and 2 line segments. It is \( f' \), the derivative of function \( f \).
Function \( f \) is defined on \([-3, 4]\) and \( f(0) = 2 \).
A = the maximum value of \( f \) on \([-3, 4]\).
B = the minimum value of \( f \) on \([-3, 4]\).
C = the volume of the solid formed by rotating the region between \( f' \) and the \( x \)-axis on \([2, 4]\) around the \( x \)-axis.
Evaluate \( \frac{C}{B + 2} + 2A \)

Question #12
Calculus Bowl 2004
If \( \frac{dy}{dx} = 2y(2x^2 - 2) \) and \( y(0) = 3 \), find \( y(2) \).

Question #13
Calculus Bowl 2004
Find the sum of the \( y \)-intercepts of the lines which are tangent to the graph of \( xy = 4 \) and contain the point \((3, 1)\).

Question #14
Calculus Bowl 2004
If \( \frac{dy}{dx} = [2x + 1] \), find the average rate of change of \( y \) with respect to \( x \) on the interval \([0, 4]\).

Question #15
Calculus Bowl 2004
The region bounded by the \( x \)-axis and the part of the graph of \( y = x^2 \) between 0 and 4 is separated into two regions by the line \( x = p \). If the area of the region on \([0, p]\) is 1 square unit less than the area of the region on \([p, 4]\), find the value of \( p \).