

CALCULUS INDIVIDUAL  
FAMAT State Convention 2004

For all questions, E. NOTA means none of the above answers is correct.

1. Evaluate  $\lim_{x \rightarrow 0} \frac{0.5 \sin(2x) + x \sin x - e^x + 1}{96x \sin x}$

- a. 0
- b. 1/100
- c. 1/200
- d. 1/192
- e. NOTA

2. Given that  $f$  is continuous on  $[2, 5]$ ,  $f(2) = -1$ ,  $f(4) = 2$ , and  $f(5) = -3$ , which of the following must be true?

- I.  $f(c) = 1$  for some  $c$  in  $(2, 5)$ .
- II.  $f'(c) = 0$  for some  $c$  in  $(2, 5)$
- III.  $f$  has at least two zeros in  $(2, 5)$

- a. I, II, and III
- b. I and II only
- c. II and III only
- d. I and III only
- e. NOTA

3. An object is dropped from a point 600 feet above the ground. Its position at time  $t$  seconds after it is dropped is given by  $f(t) = 600 - 16t^2$ . With what velocity in ft/sec does it strike the ground?

- a.  $-80\sqrt{6}$
- b.  $-40\sqrt{6}$
- c.  $-20\sqrt{6}$
- d. -196
- e. NOTA

4. Evaluate  $\lim_{x \rightarrow 5^+} \frac{\sqrt{7x-10} - 5}{10x - 50}$

- a. 0
- b. 0.07
- c. 0.001
- d. 0.007
- e. NOTA

5. Find the abscissas of points on the graph of  $y = x^3 + 2x^2 - 4x + 5$  at which the tangent line is parallel to  $11x - y = 4$ .

- a.  $\left\{-2, \frac{2}{3}\right\}$
- b.  $\left\{\frac{5}{3}, -3\right\}$
- c.  $\left\{\frac{5}{2}, -2\right\}$
- d.  $\phi$
- e. NOTA

6. Given  $f$  and  $g$  are twice differentiable functions,  $f(2) = 3$ ,  $f'(2) = -1$ ,  $g(2) = 2$ ,  $g'(2) = -2$  and  $f(g(x)) = h(x)$  for all  $x$ . Find  $h''(2)$  if  $f''(2) = g''(2)$ .

- a.  $3f''(2) - 4$
- b.  $f''(2)$
- c.  $3f''(2)$
- d. 0
- e. NOTA

7. If  $\frac{dy}{dx} = \sqrt{2x+1}$  find the average rate of change of  $y$  with respect to  $x$  on the interval  $[0, 4]$ .

- a. 13/6
- b. 26/3
- c. 52/9
- d. 6
- e. NOTA

8. Use differentials to approximate the increase in the number of cubic inches in the volume of a spherical balloon if the diameter changes from 4 inches to 4.02 inches. Give the answer to the nearest thousandth.

- a. 4.021
- b. 2.011
- c. 1.005
- d. 0.503
- e. NOTA

9. If  $w = \frac{z^2 - 27}{27z}$  and  $z = (x^2 + 2)^3$ , find  $\frac{dw}{dx}$  when  $x = 1$ .

- a. 56/27
- b. 7/3
- c. 8/3
- d. 65/27
- e. NOTA

10. Find  $\frac{d^2y}{dx^2}$  at  $y = 3$  if  $\frac{dy}{dx} = \sqrt{2y^2 - 1}$ .

- a.  $\frac{\sqrt{17}}{34}$
- b.  $\frac{6\sqrt{17}}{17}$
- c. 6
- d. 4
- e. NOTA

11. At the point (0,0) the graph of  $f(x) = e^x + e^{-x} + 2 \cos x - 4$  has:

- a. a relative maximum
- b. a relative minimum
- c. an inflection point
- d. a discontinuity
- e. NOTA

12. Find the maximum value of  $y = \cos(2x) + \sin(2x)$ .

- a. 1
- b.  $2\sqrt{2}$
- c.  $\sqrt{2}$
- d.  $\frac{\sqrt{2}}{2}$
- e. NOTA

13. Let  $f$  be a function that is continuous on  $[a, b]$ . Which are necessarily true if  $f(a) > 0$  and for all  $x$  in  $[a, b]$ ,  $f'(x) < 0$  and  $f''(x) < 0$ .

- I. For all  $c$  and  $d$  in  $(a, b)$  such that  $a < c < d < b$ ,  $f(d) < f(c) < f(a)$ .
- II. There exists an  $x$  in  $(a, b)$  such that  $f'(x) > f'(b)$ .
- III.  $f$  has at least one zero in  $(a, b)$ .
- IV. For some  $w$  in  $(a, b)$ ,

$$f''(w) = \frac{f'(b) - f'(a)}{b - a}$$

- a. all are true
- b. all except III are true
- c. I and II only are true
- d. I only is true
- e. NOTA

14. Evaluate

$$\lim_{n \rightarrow \infty} \left( \frac{\pi}{n} \left( \sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \sin \frac{3\pi}{n} + \dots + \sin \frac{n\pi}{n} \right) \right)$$

- a. 2
- b.  $\pi$
- c.  $1 - \cos 1$
- d. 0
- e. NOTA

15. If  $g(x) = \int_3^x \frac{t}{e^t + 1} dt$  evaluate  $g''(0)$ .

- a. 2
- b. 1
- c.  $\frac{1}{4}$
- d. 0
- e. NOTA

16. Which of the following is equivalent to

$$\int_1^2 \frac{x}{e^{x^2}} dx ?$$

- a.  $\int_1^2 e^{-u} du$
- b.  $\int_1^4 e^{-u} du$
- c.  $\frac{1}{2} \int_1^2 e^{-u} du$
- d.  $\int_1^4 \frac{e^{-u}}{2} du$
- e. NOTA

17. Evaluate  $\int_0^{\pi} |\sin x - \cos x| dx$

- a.  $\sqrt{2}$
- b. 2
- c.  $2\sqrt{2}$
- d. 4
- e. NOTA

18. Suppose  $y = f(x)$  is a differentiable function such that  $f(2) = 0$  and such that  $3x^2 - x^2y^3 + 4y = 12$ . Use this value of  $f(2)$  to determine a differential approximation for  $f(1.97)$ .

- a. 0.09
- b. -0.09
- c. .97
- d. -1.06
- e. NOTA

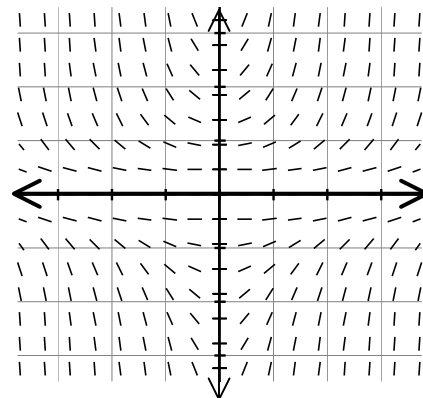
19. The function  $v(t) = \frac{\sin t}{\sqrt{t}}$  for  $t$  in the

interval  $(0, 4]$  describes the velocity of a particle moving on the  $x$ -axis. Which of the following is true at  $t = 3.5$ ?

- a. acceleration is negative and speed is increasing
- b. acceleration is negative and speed is decreasing
- c. acceleration is positive and velocity is increasing
- d. acceleration is negative and velocity is increasing.
- e. NOTA

20. Choose the differential equation that could produce the given slopefield?

- a.  $\frac{dy}{dx} = \frac{xy^2}{2}$
- b.  $\frac{dy}{dx} = x + \frac{y^2}{2}$
- c.  $\frac{dy}{dx} = \frac{x^2y}{2}$
- d.  $\frac{dy}{dx} = x + y$
- e. NOTA



21. Which of the following is true regarding the graph of  $y = 5x^4 + 3x^5$ ?

- a. It has two inflection points and two local extrema.
- b. It has one inflection point and two local extrema.
- c. It has two inflection points and one local extrema.
- d. It has one inflection point and one local extrema.
- e. NOTA



22. Evaluate

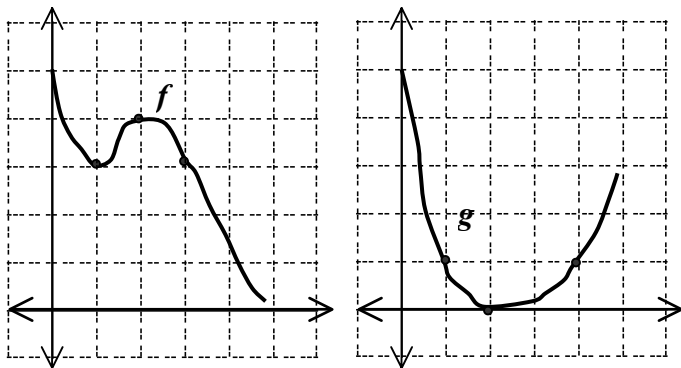
$$\int_0^4 f(x)dx \text{ if } f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 2x-4 & \text{for } 3 < x \leq 4 \end{cases}$$

- a.  $29/3$
- b. 9
- c.  $25/3$
- d.  $23/3$
- e. NOTA

23. If the graph of  $y = 3x^2 + 2x + k$  is tangent to the line  $4x - y = 3$ , find  $k$ .

- a. 6
- b. -3
- c.  $-7/3$
- d.  $-2/3$
- e. NOTA

24. Graphs of  $f$  and  $g$  are shown. If  $h(x) = f(g(x))$ , which of the following statements could be true based on these graphs?



NOTE: Scales on all axes are 1.  
 $f$  has local extrema at  $x=1$  and at  $x=2$ ,  $g$  has a local extremum at  $x=2$ .

- I.  $h(2) = 3$
- II.  $h$  is increasing at  $x = 3$
- III.  $h$  has a horizontal tangent line at  $x = 1$ .
- IV.  $h$  is concave up at  $x = 4$

- a. I and II only
- b. III and IV only
- c. II and IV only
- d. I and III only
- e. NOTA

25. Water is flowing into a conical reservoir with diameter 12 feet and height 8 feet. If water flows in at  $6\pi \text{ ft}^3 / \text{min}$  at what rate is the surface area increasing in  $\text{ft}^2 / \text{min}$  when the height of the water reaches 4 feet high from the vertex.

- a.  $3\pi$
- b.  $4\pi$
- c.  $6\pi$
- d.  $8\pi$
- e. NOTA

26. Evaluate  $\int_0^2 (xf'(x^2) + 1)dx$  if

$$f(0) = 2, f(2) = 4, \text{ and } f(4) = 14.$$

- a. 5
- b. 6
- c. 7
- d. 8
- e. NOTA

27. If  $f(x) = \ln(2x+1)$ , find  $f(0) + f'(0) + f''(0)$ .

- a. -1
- b. -2
- c. 6
- d. 2
- e. NOTA

28. If  $f(x) = xe^{-kx}$  where  $k > 0$ , on what interval(s) is the graph of  $f$  concave down?

- a.  $(0, \infty)$
- b.  $\left(\frac{1}{k}, \infty\right)$
- c.  $\left(-\infty, \frac{2}{k}\right)$
- d.  $\left(0, \frac{2}{k}\right)$

- e. NOTA

29. If  $f(x) = 2x^3 + 3x^2 - 12x + 7$ , find the sum of the values of  $f''$  at each distinct real zero of  $f$ .

- a. 0
- b. -18
- c. 36
- d. 48
- e. NOTA

30. Write an equation of a line tangent to the graph of  $f(g(x))$  at  $x = -1$  if

$$f(x) = 2x^3 + x^2 - x + 1 \text{ and}$$

$$g(x) = x^5 - 4x^3 + 2x - 3.$$

- a.  $19x - y = -10$
- b.  $95x - 5y = -102$
- c.  $19x + 2y = -54$
- d.  $95x + y = -104$
- e. NOTA