Integration Test - Calculus — FAMAT State 2004

a 1. $\sqrt{2}\int_2^3 \frac{1}{|x|}\,dx = \sqrt{2}\,|\ln x|_2^3 = \sqrt{2}\ln\frac{3}{2}$

b 2. $x^2 = 2x+3$ $(x-3)(x+1)=0$ $x=3$ $x=-1$ $\int_{-1}^{3}(2x+3-x^2)\,dx = \left|x^2+3x-\frac{x^3}{3}\right|_{-1}^{3} = (9+9-9)-\left(1-3+\frac{1}{3}\right) = \frac{32}{3}$

c 3. $\int_1^3 (x^3-6x^2+11x-6)\,dx = 0$

d 4. $\int e^{-x}\,dx + \int \frac{1}{x}\,dx = -e^{-x}+\ln|x|+C$

c 5. $h = \left(4-\frac{x}{2}\right)$ $\frac{\pi}{2}\int_0^4 \left(4-\frac{x}{2}\right)^2 = \frac{\pi}{8}\int_0^4 (16-8x+x^2)\,dx = \frac{\pi}{8}\left|16x-4x^2+\frac{x^3}{3}\right|_0^4 = \frac{\pi}{8}\cdot\frac{64}{3} = \frac{8\pi}{3}$

d 6. $u=\sqrt{x}$ $du = \frac{1}{2\sqrt{x}}dx$ $= 2\int \sec u\,du = 2\ln|\sec u+\tan u|+C$ $2\ln|\sec\sqrt{x}+\tan\sqrt{x}|+C$

a 7. $u=\sqrt{4x+1}$ $u^2=4x+1$ $2u\,du = 4\,dx$ $u/2 = dx$ $3\int_1^3 \frac{u^2-1}{4}\cdot u\cdot\frac{u}{2}\,du = \frac{3}{8}\int_1^3 (u^4-u^2)\,du$

a 8. $y^2dy = \frac{x-1}{x}dx$ $\frac{y^3}{3} = x-\ln|x|+C$ $\frac{8}{3}=1+C$ $C=\frac{5}{3}$ $\frac{y^3}{3} = x-\ln|x|+\frac{5}{3}$ $y(2) = \sqrt[3]{(10-3\ln 2)3}$

c 9. $\pi\int_0^{\pi/4}\sec^2 x\,dx = \pi\left|\tan x\right|_0^{\pi/4} = \pi(1-0) = \pi$

a 10. i (T) ii (F) iii (T) iv (F) (d) 11. $\int_0^1 e^{2x}\,dx = \frac{1}{2}\int_0^2 e^u\,du = \frac{e^2-1}{2}$

c 12. $\frac{1}{b}\int_0^b (2+6x-3x^2)\,dx = 3$ $\left|2x+3x^2-x^3\right|_0^b = 3b$ $b(b^2-3b+1)=0$ $b^2-3b+1=0$ $\frac{3\pm\sqrt{9-4(1)(1)}}{2}$ $\frac{3\pm\sqrt{5}}{2}$

c 13. $\pi\int_0^1 \left[\tan(x^2)\right]^2 dx$ $\pi\left[.25\left[\tan^2\left(\frac{1}{4}\right)+\tan^2\left(\frac{2}{4}\right)+\tan^2\left(\frac{3}{4}\right)+\tan^2\left(\frac{4}{4}\right)\right]\right] = 0.875$

a 14. $x\cos(\pi x)\pi + \sin(\pi x) = 2xS(x^2)$ $x=2$ $2\pi = 4S(4)$ $S(4) = \frac{\pi}{2}$

a 15. $e^{x^2}g'(x)+g(x)2xe^{x^2} = 2xe^{x^2}g(x)$ $g'(x)+2xg(x) = 2xg(x)$ $2x\,g(x) = 2xg'(x)-g'(x)$ $\frac{dy}{2x-1} = \frac{g'(x)}{g(x)}$ $\int\frac{dx}{2x-1} = \int\frac{g'(x)}{g(x)}$ $x+\frac{1}{2}\ln|2x-1|+C = \ln|g(x)|+C$ $g(x) = e^{x+\frac{1}{2}\ln(2x-1)+C}$ $g(x) = Ce^x\sqrt{2x-1}$

c 16. $\frac{1}{10}\int_0^{10} 30{,}000\,e^{-0.13t}\,dt = -0.13$ $\frac{30{,}000}{-.13}\int_{1.3}^{1.3} 2^u\,du = \frac{30{,}000}{-.13}\left|\frac{2^u}{\ln 2}\right|_{1.3}^{1.3}$

c 17. $a=1$ $u=x^2$ $du=2x$ $\int_{1/2}^{\sqrt{3}/2}\frac{du}{\sqrt{1-u^2}} = \left|\arcsin u\right|_{1/2}^{\sqrt{3}/2} = \frac{\pi}{3}-\frac{\pi}{6} = \frac{\pi}{12}$

a 18. $\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$ $S = 4\pi r^2$ $4\pi r^2\frac{dr}{dt} = k(4\pi r^2)$ $\frac{dr}{dt} = k$ $\frac{dr}{dt} = -k$ $r = -kt+C$ $3=C$ $r = -kt+3$ $1 = \frac{-2}{5}k+3$ $k = \frac{2}{5}$ $r = -\frac{2}{5}t+3$ $r(7) = 1/5$

d 19. $\int_1^5 \frac{t^2}{3+t^3}\,dt$ $u = 3+t^3$ $\frac{du}{dt} = 3t^2$ $\frac{1}{3}\int_3^{128}\frac{1}{u}\,du = \frac{1}{3}\left[\ln|128|-\ln 3\right]$

c 20. $x = ax^2$ $0 = x(ax-1)$ $\int_0^{1/a}(x-ax^2)\,dx = \frac{4}{5}$ $\left|\frac{x^2}{2}-\frac{ax^3}{3}\right|_0^{1/a} = \frac{4}{5}$ $\frac{1}{2a^2}-\frac{1}{3a^2} = \frac{4}{5}$ $\frac{1}{6a^2} = \frac{4}{5}$ $\frac{2}{6a^2}-\frac{2}{6a^2} = \frac{4}{5}$ $a = \sqrt{5/24}$

b 21. $v(t) = -\frac{1}{2}\cos(2t)+C$ $v(t) = -\frac{1}{2}\cos(2t)+3$ $x(t) = -\frac{1}{4}\sin(2t)+3t+C$ $x(t) = -\frac{1}{4}\sin(2t)+3t+\frac{25-3\pi}{4}$ $x(\pi/2) = \frac{25+3\pi}{4}$

c 22. $2\pi\int_0^k \frac{x}{4+x^2}dx = \pi\ln 2$ $u = 4+x^2$ $du = 2x\,dx$ $\int_4^{k^2+4}\frac{1}{u}\,du = \ln 2$ $\ln\frac{k^2+4}{4} = \ln 2$ $\frac{k^2+4}{4} = 2$ $k=2$

a 23. $\int_0^a 2x\,dx = \left|x^2\right|_0^a = a^2$ $y = \sqrt{a-x^2}$ $\int_0^a \sqrt{a-x^2}\,dx = \frac{\pi a^2}{4}$ $a^2+\frac{\pi a^2}{4}$ $\frac{4a^2+\pi a^2}{4}$

b 24. $14-31x-10x^2=0$ $(2-5x)(7+2x)=0$ $x=\frac{2}{5}$ $x=-\frac{7}{2}$ when $a=-7/2$ $b=2/5$ Max area occurs

a (T) b (F) c (T) 25. i. (T) ii (F) iii (T) iv (F) v (T) vi (F)

d 26. $2\sin A\sin B = -\cos(A+B)+\cos(A-B)$ $4\sin x\sin 8x = 2\left[2\sin x\sin 8x\right] = 2\left[-\cos(9x)+\cos(-7x)\right]$

b 27. $\int_{\pi/6}^{\pi/4}\cot x\,dx = \left|\ln|\sin x|\right|_{\pi/6}^{\pi/4} = \ln\left(\frac{\sqrt{2}}{2}\right)-\ln\frac{1}{2} = \ln\sqrt{2} = \frac{\ln 2}{2}$

b 28. $\int(\sin^2x+2\sin x\cos x+\cos^2x)\,dx = \int(1+\sin(2x))\,dx = x-\frac{\cos(2x)}{2}+C$

d 29. $\sqrt{x} = \sin y$ $x = \sin^2 y$ $dx = 2\sin y\cos y\,dy$ $1-x = 1-\sin^2 y$ $\cos^2 y$ $\int_0^{\pi/2}\frac{\sin y\cdot 2\sin y\cos y\,dy}{\cos y} = \int_0^{\pi/4} 2\sin^2 y\,dy$

a 30. $x = u^{\frac{n+1}{n}}$ $u = \sqrt[n+1]{x}$ $\int_0^2 (x-1)^{1/n}\,dx = \int_{-1}^{1} u^{\frac{1}{n}}\,du = \left|u^{\frac{n+1}{n}}\right|_{-1}^{1}$ n odd is even $\therefore (1-1)=0$