# Limits and Derivatives
# Mu Alpha Theta State Convention 2004
**Note: The answer choice E (NOTA) stands for "none of the above"**

1. If a particle moves with position defined by $x(t) = 8t^4 - 2t^2 + 9$ for $t \geq 0$, what is the rate of change of its acceleration at $t = 1$?

A. 92 B. 132 C. 168 D. 192 E. NOTA

2. Calculate $\lim_{x \to 0} \cot(7x)\sin(4x)$.

A. $\frac{7}{4}$ B. $\frac{4}{7}$ C. 0 D. Does Not Exist E. NOTA

3. Find the Limit: $\lim_{x \to \infty} \frac{a + bx^3}{c - dx^3}$

A. $\frac{a}{c}$ B. $\frac{b}{d}$ C. $\frac{a}{d}$ D. $\frac{b}{c}$ E. NOTA

4. Given that $f(-x) = f(x)$ and that $f'(x)$ exists for all x, what must $f'(-a)$ equal?

A. $f'(a)$ B. $-f'(a)$ C. $\frac{-1}{f'(a)}$ D. $\frac{1}{f'(a)}$ E. NOTA

5. Suppose we have N bingo balls, each numbered 1 to N. What is the probability that randomly choosing one of these balls, we never pick the ball #1, if we attempt this experiment N times, replacing the ball we remove each time, as N goes to infinity? (round to the nearest thousandth)

A. 0.001 B. 0.126 C. 0.368 D. 0.277 E. NOTA

6. How many inflection points does the function $f(x) = \frac{1}{4}x^4 - 3x^3 + 2x^2$ have?

A. 1 B. 2 C. 3 D. 4 E. NOTA

7. $\frac{d}{dx}\int_1^{x^3} t^2 \sin t \, dt$?

A. $x^2 \sin x$ B. $x^6 \sin x^3$ C. $3x^3 \sin x^3$ D. $3x^8 \sin x^3$ E. NOTA

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8. Given $3x^2y^2 + 5y^3x = 128$, what is the value of $\frac{dy}{dx}$ when $y = 2$ and $x = 2$?

A. $\frac{11}{21}$ B. $\frac{-11}{21}$ C. $\frac{-21}{11}$ D. $\frac{21}{11}$ E. NOTA

9. If $y = \frac{1}{\sqrt{x^3 + 1}}$, what is $y^{(2)}(2)$?

A. $\frac{2}{9}$ B. $\frac{-2}{9}$ C. $\frac{-6}{11}$ D. $\frac{7}{11}$ E. NOTA

10. If $g(x) = \sin x$, find the value *c* on the interval $[0, \pi]$ that satisfies the mean value theorem for derivatives.

A. $\frac{\pi}{6}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{2}$ D. $\frac{3\pi}{2}$ E. NOTA

11. The We Say So Corporation™ is constructing a circle and square out of floss in hopes of creating better flossing devices for the LAC (Large Animal Community), which will bring the company millions if done properly. If you are given 12 ft. of floss, how much should be used for each shape to yield the maximum area (circle and square, respectively), and thus the maximum flossing capacity? (round to the nearest hundredth)

A. 6.72 ft; 5.28 ft B. 5.72 ft; 6.28 ft
C. 6.28 ft; 5.72 ft D. 5.28 ft; 6.72 ft E. NOTA

12. If $f(x) = 2x^5 + x^3 + 1$ and $g(x) = f^{-1}(x)$, then find $g'(-2)$.

A. $\frac{1}{14}$ B. $-14$ C. $14$ D. $173$ E. NOTA

13. Find $r'(x)$ given $r(x) = \arctan(\frac{3}{5}x)$.

A. $\frac{5}{9x^2 + 25}$ B. $\frac{15}{9x^2 + 25}$ C. $\frac{3}{9x^2 + 25}$ D. $\frac{9}{9x^2 + 25}$ E. NOTA

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14. Which of the following functions are not differentiable at $x = 0$?

I. $3x+1$ II. $x^{2/3}-2$
III. $|2x-1|$ IV. $[[x]]$ (Greatest Integer Function)

A. I,II,III B. I, III, IV C. II, IV D. II, III, IV E. NOTA

15. $\lim\limits_{m \to 3} \dfrac{m^4 - 18m^3 + 116m^2 - 318m + 315}{m^2 - 6m + 9} = ?$

A. 20 B. 14 C. 8 D. 0 E. NOTA

16. $\lim\limits_{x \to 1} x^{\frac{1}{x-1}} = ?$

A. 0 B. 1 C. $e$ D. DNE E. NOTA

17. $\lim\limits_{x \to \infty} \dfrac{\sqrt{4x^2+2}}{3x+1} = ?$

A. $\dfrac{4}{3}$ B. $\dfrac{2}{3}$ C. 0 D. DNE E. NOTA

18. On what intervals is $x^3 - x + 5$ increasing?

A. $(-\infty, \infty)$ B. $(-\infty, \dfrac{1}{\sqrt{3}}]$

C. $(-\infty, \dfrac{-1}{\sqrt{3}}]$ U $[\dfrac{1}{\sqrt{3}}, \infty)$ D. $[\dfrac{1}{\sqrt{3}}, \infty)$ E. NOTA

19. Let $a(n) = \sum\limits_{i=1}^{n} \left(3 + \dfrac{2i}{n}\right)^2 \left(\dfrac{1}{n}\right)$. Find $\lim\limits_{n \to \infty} a(n)$.

A. $\dfrac{53}{3}$ B. $\dfrac{49}{3}$ C. $\dfrac{44}{3}$ D. $\dfrac{40}{3}$ E. NOTA

20. If $f(x) = \cos(x) - \sin(x)$, what is the value of $\sum\limits_{i=0}^{2004} f^{(i)}(\pi)$?

A. 0 B. 1 C. 2004 D. -2004 E. NOTA

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21. $$f(x) = \begin{cases} k & x = 3 \\ \dfrac{2x^3 + 3x^2 - 32x + 15}{(x-3)} & x \neq 3 \end{cases}$$

For what value of $k$ will $f(x)$ be continuous for all values of $x$?

A. 3  B. 40  C. 90  D. 0  E. NOTA

22. Using differentials and the function $\dfrac{1}{\sqrt[3]{2x+3}}$, estimate $\dfrac{1}{\sqrt[3]{31}}$ given $\dfrac{1}{\sqrt[3]{27}} = \dfrac{1}{3}$. (round to the nearest thousandth)

A. 0.320  B. 0.318  C. 0.308  D. 0.300  E. NOTA

23. $\dfrac{d^3}{dx^3}\left[a^{bx}\right] = ?$ ($a$ and $b$ are constants)

A. $b^3 a^{bx}$  B. $(ab)^3 a^{bx}$  C. $\left(\dfrac{b}{a}\right)^3 a^{bx}$  D. $\left(\dfrac{1}{ab}\right)^3 a^{bx}$  E. NOTA

24. A 25 $ft$ ladder leans against a wall, with its top being 15 $ft$ from the ground. This is one of many faulty ladders in today's ladder market, a problem so profound that I can't even begin to explain on this test. Thus, it has no grip on either end. If the top of the ladder falls at a rate of 1.5 $ft/hour$, at what rate does the bottom of the ladder move away from the wall? (round to the nearest hundredth)

A. 1.13 $ft/hour$  B. 1.12 $ft/hour$
C. 1.51 $ft/hour$  D. 1.50 $ft/hour$  E. NOTA

25. Find $\lim\limits_{x\to 4} \dfrac{d}{dx}\left[(x-4)^2 g(x)\right]$ where $g(x) = \dfrac{x^3 - 4x^2 + 2x - 1}{(x-4)^2}$ (In complex analysis, this is known as the residue of $g(x)$ at $x = 4$).

A. 25  B. 7  C. 4  D. DNE  E. NOTA

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26. Find the point on the graph $y = 3 - 5x^2$ that is closest to the point $(1, 0)$.

A. $\left(\frac{9}{11}, \frac{-42}{121}\right)$ B. $\left(\frac{8}{13}, \frac{187}{169}\right)$ C. $\left(\frac{5}{12}, \frac{307}{144}\right)$ D. $(1, -2)$ E. NOTA

27. $\lim_{k\to\infty} \sum_{n=0}^{k} \frac{(-2)^n}{(2n+1)!}$ = ?

A. $\frac{\sin(\sqrt{2})}{\sqrt{2}}$ B. $\frac{\pi}{2}$ C. $e^{2\pi}$ D. $\frac{\cos(\sqrt{2})}{\sqrt{2}} - 1$ E. NOTA

28. The movie "Pirates of the Arctic Ocean" has just come out on DVD and is making a huge profit, with a cost function of $C(x) = 0.89x + 10000$ and revenue function of $R(x) = x^2 + 4000$, where $x$ represents the number of copies sold. What is the marginal profit for this company after they've sold one thousand copies? (round to the nearest hundredth)

A. 1999.11 B. 1989.11 C. 1827.11 D. 1888.11 E.NOTA

29. $\lim_{x\to 4} \frac{\sqrt{x+5} - 3}{x - 4} = ?$

A. $\frac{1}{3}$ B. $\frac{1}{6}$ C. $\frac{1}{4}$ D. DNE E. NOTA

30. A population grows according to the equation $P(t) = 8000 - 4300e^{-0.178t}$ for $t \geq 0$ where $t$ is measured in years from January 1, 2004. As time goes on and $t$ gets infinitely large, this population will reach a limiting value. During which year will the population reach 75% of this limiting value?

A. 2006 B. 2007 C. 2008 D. 2009 E. NOTA