1. The arrow is made by the letters 'E' and 'x' and it points to the **right** of the logo.

2. I-95…~382; I-295…~35; I-595…~12; I-195…~5; I-275…~62; I-75…~469; I-10…~363; I-110…~6; I-4…~133. Total: ~1467…will take any answer within 20 miles of this answer.

3. Two squares

4. Rectangles

5. Not too difficult a problem if you right out the six digits one at a time…you will see a pattern. I started off with 012345…and started counting up. The total number is 210.

6. 2-$500, 2-$100, 2-$50, 6-$20, 5-$10, 5-$5, 5-$1 for a total of 27 bills.

7. 85 and 86. These correspond to 30 degrees Celsius, if you consider that it can range from 29.50 to 30.49.

8. The biggest factor in making the most money is trying to get the most Sunday hours worked. To do this she should work the R shift (2 PM to 10 PM) on both Sundays. Then after that she can work the X shift (10 PM to 6 AM) during the rest of the week to get the 10% night bonus each night. Thus the most money can be made working a shift schedule of: R/Off/X/X/X/X/X/R/Off/X/X/X/Off/Off. This gets her 8 hours at $27/hour, 8 hours at $25/hour, and 64 hours at $22/hour, which adds up to $1824. To make the least amount, she wants to work the most day/morning shifts possible within the rules. She can do this by throwing in an R shift in between a long string of G shifts (6 AM to 2 PM)…or Off/G/G/G/G/R/Off/Off/G/G/G/G/G/Off. This gets her 76 hours at $20/hour and 4 hours at $22/hour, which adds up to $1608. So the range is $216.

9. The easiest way to solve this problem is to go to a banks web page and look for a loan calculator. Put the total in, the interest rate, and the new term of the loan as 120 months. This will get you the new monthly rate of $1341.17…or a difference of $341, when rounded to the nearest dollar. The only thing to check is that they compound the interest monthly. I will also accept answers from $341 to $350, as any of these amounts over the original $1000 will make the new term 120 months.

10. First make sure and convert everything into feet and seconds. So the balloon rises at 16.4042 ft/s. The wind increases at 7.333244 ft/s per 1000 ft. You want to find out when z/x=tan 5. Based on this information, you can get equations for z and x as functions of t. $z(t)=16.4042t$. $v(t)=(7.333244/1000)*z(t)=.120296t$. Integrating for x gives $x(t)=.060148t^2$. So z/x=272.7306/t=tan5, which leads to t=3117.325 s, or 52 minutes.

11. At the time t above, z is 51,137 feet, or 51,000 feet.

12. You can add up the first and third rows to create a new third row. Then add up the first and fourth rows to create a new fourth row. Then add up the second and two times the new fourth row to create a new fourth row. Then rows three and four are the same, and that leaves a rank of 3 to the matrix.

13. The speed of light is 2.99792×10^8 m/s. One year is 3.1536×10^7 s. So the distance is $x=(2.99792 \times 10^8 \text{ m/s})(4.2)(3.1536 \times 10^7 \text{ s})=3.9708 \times 10^{16} \text{ m}=2.5 \times 10^{13}$ miles.
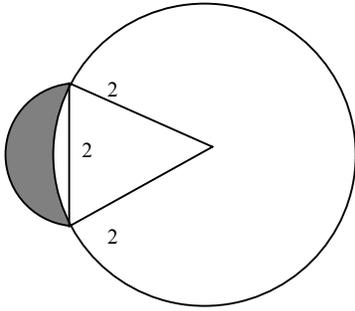
14.   The area of the lune is determined by finding the area of the equilateral triangle topped by the small semicircle and then subtracting the area of the sector of the larger circle. The former has an area of $\pi/2$+root 3. The area of the latter is $2\pi/3$. The difference is $-\pi/6$+root 3.

15. Rearrange the equations to read a+12c=9+8b and 12a-c=8-9b. Then square both equations and add them together to get $145(a^2+c^2)=145(1+b^2)$. So the two terms in parentheses must be equal, and $a^2-b^2+c^2=1$.

16. By setting n=1 in the given relation, you obtain $a_{m+1}= a_m+1+m$, for all positive integers m. So $a_{m+1}-a_m=m+1$. Hence $a_{2004}-a_{2003}=2004$, $a_{2003}-a_{2002}=2003$, …$a_2-a_1=2$. Adding all of these equations gives $a_{2004}-1=2004+2003+…+3+2$. So $a_{2004}=2004(2005)/2=2009010$.

17. The standard deduction for a single person is $4,750. Subtract this from his income, and you get $15,250. Subtract the exemption for himself of $3,050, and he is left with $12,200. Looking at the tax tables, this means his tax is $1,484. Since he paid $1,000 by withholding money from his paychecks, he owes $484.

18. She was struck by the iceberg around 1145 PM on April 14, but she foundered on April 15 of 1912. If you look up old newspaper reports, you will see that this date is a Monday.

19. 3,225 years (Ref: http://www-astronomy.mps.ohio-state.edu/~peterson/Ast291/ notes09.pdf)

20. This is the Fibonacci Sequence, and as *n* goes to infinity, consecutive terms tend toward the Golden Ratio, or $\left(\sqrt{5}+1\right)/2$.

21. From the equation a+b+c=-(.2)/3, ab+ac+bc=-3/3=-1, and abc=-(-.1)/3.
Now of $a^3+b^3+c^3=(3/2)[(a+b+c)^2-2(ab+ac+bc)](a+b+c)-(1/2)[(a+b+c)^3-6abc]$
       $=(3/2)[(-.2/3)^2-2(-1)](-.2/3)-(1/2)[(-.2/3)^3-6(.1/3)]=-677/6750$

22. BD is trisected by AQ and AP. So we need to find the length of BD and then divide it by 3. If we call angle D θ, then, by the cosine law $12^2=8^2+10^2-2(8)(10)\cos\theta$ or $\cos\theta=1/8$. Now angle A is 180 minus angle D. Again by the cosine law, $BD^2=8^2+10^2-2(8)(10)\cos(180-\theta)=164+160\cos\theta=184$. So the length of EF is (2/3)(root 46).

23. To find the number of digits in a number, you must take the log (base 10) of that number, add 1, and then truncate to an integer. For example, the number 27: log 27=1.43…, adding 1 and truncating makes it 2. The number 27 has 2 digits. Log(2004!)=log 2004+log 2003+… Summing it, adding 1, and truncating gives you 5749.

24. "I must study politics and war that my sons may have liberty to study mathematics and philosophy." Quoted by John Adams in a letter to Abigail Adams on May 12, 1780.

25. $2004^2 \bmod 10000 = 6016 \ldots 2004^4 \bmod 10000 = (2004^2 \bmod 10000)^2 \bmod 10000 = 2256$. Continuing on in this way (multiplying the mods of other numbers), you eventually get to $2004^{2004} \bmod 10000 = 2256$. Next we need to determine what $2256^{2004} \bmod 10000$. This is, by the same method as above, 5296.

26. The first number is the 12th number in the Fibonacci sequence. The next number is the 14th, then the 16th, …The number to fill the sequence would then be the 22nd, or 17711.

27. $(600 \text{ drops}/10^9 \text{ mm}^3)[(\pi/6)(1 \text{ mm})^3/1 \text{ drop}](4 \text{ m/s}) = 1.25664 \times 10^{-6} \text{ m/s} = 0.18 \text{ in/hr}$

28. Plugging the R of 0.18 in/hr into the formula gives a Z of 27.196. Converting this to dBZ gives 14.

29. You can break up this sequence of numbers into three different sets of sequences: 1. 1, 1/2, 1/4, 1/8, …, 2. 2, -2/3, 2/9, -2/27, …, and 3. Split infinitely by smaller values…-1, -1/4, -1/16… added to another series that starts in the second position with -1/4, -1/16…, added to another series that starts in the third position with -1/16…, and this goes on to infinity. The sum of the first set is 1/(1-1/2)=2. The sum of the second is 2/[1-(-1/3)]=3/2. The sum of the list is itself a series, starting with -1/(1-1/4)=-4/3, and then –(1/4)/(1-1/4)=-1/3, and then –(1/16)/(1-1/4)=-1/12. The sum of this third set is then –(4/3)/(1-1/4)=-16/9. Adding up the three sets gets 31/18.

30. I put together a program to run through the possible five digit numbers and add up the digits. There are 4,770 numbers that match the conditions out of a total of 90,000, so the answer is 53/1000.

31. Here are the similar characters: 'B' and '8', 'D' and 'O' and 'Q' and '0', and 'S' and '5'. This reduces the character choices to 31, and $31^6 = 887,503,681$.

32. $f(x)+f(y)=x^2+6x+y^2+6y+2=(x+3)^2+(y+3)^2-16$. $f(x)-f(y)=x^2-y^2+6(x-y)=(x-y)(x+y+6)$. The given conditions can be written as $(x+3)^2+(y+3)^2 <= 16$ and $(x-y)(x+y+6)<=0$. The first inequality describes the region inside a circle of radius 4 with center (-3,-3). The second inequality can be rewritten as (x-y >= 0 and x+y+6 <= 0) or (x-y <= 0 and x+y+6 >= 0). Each of these inequalities describes a half-plane bounded by a line that passes through (-3,-3) and has a slope 1 or -1. Thus, the set R has half the area of the circle, which is 8π.

33. Math is fun and useful for life

34. 2002, 2002, all the way to the middle two number, where the 2004's continue all the way to the last number 4006. So there would be 1002 2004's, making it the unique mode. If you start out the series with 2003, it would have to end in 4007. To keep the mean 2004, the rest of the numbers in the series would have to average out to 2003. The only way to do that and keep the range at 2004 would be to have the rest of the numbers be 2003, and that would make the mode 2003, which does not satisfy the requirements of the question, so the correct answer is 4006.

35. William Shakespeare, 1564

36. 11, with the last one being Affirmed in 1978.

37. 25th, 1916 was skipped for WWI, and 1940 and 1944 were skipped because of WWII.

38. The MC of the Friday show is not Sue (clue 1), Sam (clue 4), Liz, or Ed (clue 5); it is Bill. The MC of the show that starts at 9 PM, the latest, is not Sue (clue 1), Bill (clue 3), Liz, or Ed (clue 5); it is Sam, Dr. Well (clue 4). Since Dr. Fixit, then, is not the MC of the 9 show, Ford is (clue 5), so Sam Ford is Dr. Well. Conn's show is not the last in the day or in the week (clue 5), so Conn is not Bill; nor is Conn Liz or Ed (clue 5), so she is Sue. She is not the MC of the Monday show (clue 1); neither is Hall (clue 1), Grant (clue 4), or Ford (clue 5), so it is McCoy. Liz is not the MC on Monday (clue 5), so she is not McCoy; Ed is. Sam is not the Thursday MC, for if he were, Grant would be on Friday (clue 4), and Sam would be Dr. Plant (clue 2)—which he is not (again, clue 4). So, from clue 5, Liz is the Thursday MC, Sue Conn the Wednesday MC, and Sam Ford the Tuesday MC; Bill is Dr. Fixit. Ed is Dr. Cash (clue 1). Sue is the MC earlier in the day than Ed (clue 1), Liz, and Bill (clue 5), so she is on at 5 PM. Ed is not the 8 MC (clue 1), so from clue 5, he is on at 6, Liz at 7, and Bill at 8. Bill is Grant and Liz is Dr. Plant (clue 2); her last name is Hall, and Sue, by elimination is Dr. Friend. So Dr. Plant's garden show is on Thursday at 7 PM and Liz Hall is her real name.

39. The formula for the area of a regular n-gon is $(nx^2/4)\cot(\pi/n)$. For an octagon, this is $2x^2\cot(\pi/8)$, which equals $2x^2/(-1+\text{root } 2)=x^2(2+2 \text{ root } 2)$. So the sum of a, b, and c is 6.

40. The area it can trace out is 3/4s of a circle with a radius equal to the diagonal formed by two consecutive sides of the octagon. By cosine law, it is found that $r^2=(2+\text{root2})x^2$. So the area is $(3/4)\pi(2+\text{root2})x^2$.

41. You send it to 10 people (1), they send it to 10 people…up to 100 (2), they send it to 10 people, up to 1000 (3)…so the answer is just 10^N, where N is the number of iterations, and 1 billion people makes N=9.

42. 4653[18]+729[18]+18[9]=5400.
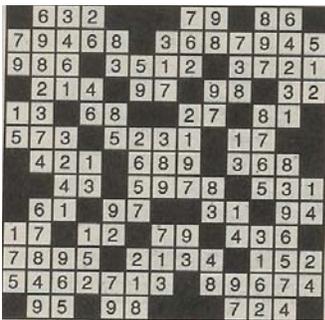
43.   The value that I want on the answer sheet is '389'.

44. The distance from the pitcher's plate to the rear point of home plate is 60.5 feet. If the release point is six feet closer, than the distance traveled is 54.5 feet. 100 mph when converted equals 146.7 feet per second. So the time is 54.5/146.7=0.37 seconds.

45. Let the common length be called L. The circumference of circle A (B) is C_A (C_B). Now $(45/360)C_A=L=(30/360)C_B$. $C_A/C_B=2/3=(2\pi r_A)/(2\pi r_B)=r_A/r_B$. Then $[\pi(r_A)^2]/[\pi(r_B)^2]=4/9$.

46. Convert the problem to base 10 and then convert back to put it in answer form. If you do this you get that $a_5=10$ and $a_{20}=70$ and, consequently, d=4, and you want to find the value of a_{15}. You can determine a_1 as -6, and then the value of a_{15} is 50. Convert this to base 7 and you have 101.

47. Kentucky…a personal favorite school of mine, and it helps that they do fairly well in basketball! As of this printing, they were ahead of North Carolina by 45 (1869, and currently 20-4, to 1824, and currently 16-8). The updated difference will be put on the answer sheet at the convention…hopefully after Big Blue wins it all!

48. Venus, Mars, Jupiter, and Saturn. The others are on the 'sunny'side of our solar system right now.

49. The highest score possible depends on the dictionary you choose. If you pick the Official Scrabble Players Dictionary, then JUKEBOX, QUIZZED, SQUEEZE, and ZYMURGY would each get you 120 points. If you use Funk & Wagnall's New Standard Dictionary, then the word ZYXOMMA would get you 130 points. I will accept any of the above answers.

50. If you start off with the exponent being only 1 you would have 4 terms, go up to an exponent of 2 and you get 10 terms, for 3 you have 20 terms, for 4 you have 35 terms. After awhile you deduce a pattern of $_{n+3}C_n$ as the answer, where the exponent is n. For n=2004, $_{2007}C_{2004}$=1,345,369,035 terms.