1. A Use the fact that the probabilities must add up to 1, and 
\[ E(X) = 1.4 = \sum_{i=1}^{4} X_i \cdot P_i(X). \]
\[ 0.4 + a + b + a + b = 1 \text{ and } a + 2b + 3(a + b) = 1.4 \Rightarrow a = 0.1, b = 0.2 \Rightarrow a^2 + b^2 = .05 \]

2. A \[ P(A \cup B) = 0.6 = 2P(A) - P(A \cap B) \text{ and } 0.8P(A) = P(A \cap B) \Rightarrow P(A) = 0.5, P(A \cap B) = 0.4 \]
\[ P[(A \cap B) \cup (\bar{A} \cap B)] = P(A \cup B) - P(A \cap B) = 1/5 \]

3. B \[ Q_3 + 1.5(Q_3 - Q_i) = 3.6 \text{ and } Q_i - 1.5(Q_3 - Q_i) = 0.5 \Rightarrow Q_i = 1.6625, Q_3 = 2.4375 \]
The median is equidistant from the two quartiles so the median equals 2.05.

4. A \[ r / r^2 = 1.25 \Rightarrow r = 0.8 \text{ and } m = r \cdot \frac{\bar{y}}{\bar{x}} \Rightarrow m = 0.4 \]
The point \((\bar{x}, \bar{y})\) lies on the regression line. Thus \(4 = 0.4(5) + b \Rightarrow b = 2\), so \(m - b = -1.6\).

5. A The sample mean is obviously 65. Thus the margin of error is 2.
\[ 2 = z * \frac{\sigma}{\sqrt{n}} = z * \frac{6}{\sqrt{20}} \Rightarrow z \approx 1.4907 \]
If we look at the standard normal table this value corresponds closely to the 86% confidence level.

6. D Let \(n\) equal the number of lifetime at-bats. The number of lifetime home runs, triples, doubles, and singles are \((0.1)(0.3)n, (0.2)(0.3)n, (0.3)(0.3)n, \) and \((0.4)(0.3)n\) respectively.
Slugging Percentage \[= \frac{\text{Total Bases}}{\text{At-bats}} = \frac{4(0.1)(0.3)n + 3(0.2)(0.3)n + 2(0.3)(0.3)n + (0.4)(0.3)n}{n} = 0.6 \]

7. D \[ P(\text{Sum of two die is prime}) = 15/36 \]
For this binomial distribution \(\mu = 300\left(\frac{15}{36}\right)\) and \(\sigma = \sqrt{300\left(\frac{15}{36}\right)\left(1 - \frac{15}{36}\right)}\).
Normal Approximation w/continuity correction \[\Rightarrow P\left(z < \frac{140.5 - \mu}{\sigma}\right) - P\left(z < \frac{99.5 - \mu}{\sigma}\right) = 0.96 \]

8. C \[ r = \frac{13 C_2 \left(\frac{4C_2 \cdot 4C_2 \cdot 44C_1}{52C_5}\right)}{13C_5} \]
Run a two-sided one-proportion z-test. \[ z = \frac{4/200 - r}{\sqrt{r(1-r)/200}} \approx -1.830 \]
This corresponds with a p-value of \(2 \times 0.0336 = 0.0672 \approx 0.067\).

9. C The survival rate for Dr. Brown’s good patients and poor patients must be higher than Dr. Benton’s.
\[ \frac{a}{68} > \frac{120}{137} \text{ and } \frac{190 - a}{182} > \frac{80}{113} \Rightarrow a > 59 \& a < 62 \Rightarrow b = 59 \& c = 62 \Rightarrow b - c = 3 \]
10. A

<table>
<thead>
<tr>
<th># of tails out of 6 coins</th>
<th>Observed</th>
<th>Expected</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>$320(\binom{6}{0}(1/2)^6) = 5$</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
<td>$320(\binom{6}{1}(1/2)^6) = 30$</td>
</tr>
<tr>
<td>2</td>
<td>86</td>
<td>$320(\binom{6}{2}(1/2)^6) = 75$</td>
</tr>
<tr>
<td>3</td>
<td>89</td>
<td>$320(\binom{6}{3}(1/2)^6) = 100$</td>
</tr>
<tr>
<td>4</td>
<td>67</td>
<td>$320(\binom{6}{4}(1/2)^6) = 75$</td>
</tr>
<tr>
<td>5</td>
<td>29</td>
<td>$320(\binom{6}{5}(1/2)^6) = 30$</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>$320(\binom{6}{6}(1/2)^6) = 5$</td>
</tr>
</tbody>
</table>

$\chi^2 \text{ Statistic} = \frac{(5 - 5)^2}{5} + \frac{(40 - 30)^2}{30} + \frac{(86 - 75)^2}{75} + \frac{(89 - 100)^2}{100} + \frac{(67 - 75)^2}{75} + \frac{(29 - 30)^2}{30}$

$\frac{(4 - 5)^2}{5} \approx 7.24$

11. B With 6 degrees of freedom and a $\chi^2$ statistic of 7.24, the p-value is greater than 0.25.

12. A Since the two variables, cholesterol before and cholesterol after, are not independent and since we can’t know the population standard deviation we must do a matched pairs t-test. In this experiment, each person is his or her own control. First, find the set of differences.

<table>
<thead>
<tr>
<th>Subject</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cholesterol After - Before</td>
<td>11</td>
<td>13</td>
<td>-8</td>
<td>-3</td>
<td>7</td>
<td>-2</td>
<td>9</td>
</tr>
</tbody>
</table>

The null hypothesis for this test is that the mean difference is 0 and the alternative hypothesis is that the mean difference is greater than 0. Because that would mean the drug has worked; that is what we are testing.

$t = \frac{\bar{x}_d - \mu_d}{s_d} \approx 1.26 \Rightarrow p - value \approx 0.13$

13. E When a one sample, two-sided z-test is performed, the critical value for $|z|$ at a 5% $\alpha$-level is 1.96. Thus the data will be significant at this level if $z$ is less than -1.96 or greater than 1.96.

$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \Rightarrow z = 1.96 < \frac{n - 130}{6 / \sqrt{10}}$ or $z = -1.96 > \frac{n - 130}{6 / \sqrt{10}} \Rightarrow n > 133.7$ or $n < 126.2$

Since $n$ must be a natural number, $c = 126$ and $d = 134$. Thus, $d - c = 8$.

14. C The primes up to 20 are 2, 3, 5, 7, 11, 13, 17, & 19. The probability that a $z$ is within 0.5 units of a prime number is

$$\frac{1}{20} \left\{ \frac{2}{5} - \frac{2}{5} + \frac{3}{5} \right\} \Rightarrow M + N = 8$$
15. C Let $p$ be the probability of getting a full house. 

$$\Pr = \binom{4}{3} \cdot \binom{4}{2} \cdot \binom{52}{5}$$

Let $X$ represent the payoff. $X$ takes two values $-1$ and $m - 1$ with probabilities of $1 - p$ and $p$ respectively. To break even in the long run, the expected value of the payoff must be 0.

$$E(X) = \sum_{i=1}^{2} X_i \cdot P_i(X) = -1 \cdot (1 - p) + (m - 1) \cdot p = 0 \Rightarrow m \approx 1388.33 \Rightarrow \log m \approx 3.1424$$

16. B There are 30 marbles total, thus $n + m = 12$. And remember $m$ must be greater or equal to than $n$.

$$P(\text{Same Color Picked}) = \frac{107}{435} \cdot \frac{10 \cdot 9 + 8 \cdot 7 + m(m-1) + n(n-1)}{30 \cdot 29}$$

Thus $m = 8, n = 4 \Rightarrow m^2 + nm + m = 100$.

17. D The probability that a certain team is not in its correct spot is $\left(1 - \frac{1}{25}\right)$. The probability that no teams are in their correct spots is $\left(1 - \frac{1}{25}\right)^{25} \approx 0.360$.

18. A The value of $\frac{n}{100} \cdot \frac{m}{100}$ represents the percentage of applicants who actually decided to attend the school. 

$$\frac{n}{100} \cdot \frac{m}{100} = \frac{1100}{9000} \Rightarrow nm \approx 1222$$

19. D The possible combinations of Cubs winning in five games are WWWLW, WWLWW, WLWWW, & LWWWW. The probabilities for these are $0.9 \cdot 0.6 \cdot 0.6 \cdot 0.4 \cdot 0.9$, $0.9 \cdot 0.6 \cdot 0.4 \cdot 0.6 \cdot 0.9$, $0.9 \cdot 0.4 \cdot 0.6 \cdot 0.6 \cdot 0.9$, and $0.1 \cdot 0.6 \cdot 0.6 \cdot 0.6 \cdot 0.9$ respectively. The sum of these probabilities is approximately 0.369.

20. C

$$\sigma_{x+y} = \sqrt{\sigma_x^2 + \sigma_y^2} \quad \text{and} \quad \mu_{x+y} = \mu_x + \mu_y \Rightarrow 10 = 4 + \tau \quad \text{and} \quad \delta^2 = \delta + \tau \Rightarrow \tau = 6, \delta = 3 \Rightarrow \ln(\tau + \delta) \approx 2.197$$

21. E For variables that follow the Poisson distribution, the standard deviation is equal to the square root of the mean.

22. C Total area under any probability density curve is 1, and $c = \frac{1}{b}, d = \frac{1}{b} + 3$

$$\frac{1}{2} \left[ b \left( \frac{1}{b} + 1 \right) - 1 \right] + b \left( \frac{1}{b} - 1 \right) + 1 = 1 \Rightarrow b = \frac{1}{2} \Rightarrow c = 2, d = 5 \Rightarrow \ln(0.5 + 2 + 5) \approx 2.0149$$
23. C In order for the class mean to go from 75 to 65 and the standard deviation to go from 8 to 5, each person’s grade must be multiplied by a number \( b \) and then a number \( c \) must then be added to each.

\[
\sigma_{\text{New}} = b \cdot \sigma_{\text{Old}} \quad \& \quad \mu_{\text{New}} = b \cdot \mu_{\text{Old}} + c \Rightarrow 5 = b \cdot 8 \quad \& \quad 65 = b \cdot 75 + c \Rightarrow b = 0.625, \ c = 18.125 \\
Andrea's \ New \ Grade = b \cdot (Andrea's \ Old \ Grade) + c \Rightarrow Andrea's \ New \ Grade = 74.375
\]

24. C Probability of a Type I error is equal to the significance level, \( \alpha = 0.05 \). Probability of a Type II error is equal to 1 minus the power of the test, \( \beta = 1 - 0.86 = 0.14 \). So \( \alpha + \beta = 0.19 \).

\[
\mu_4 = 4 \cdot 190 \quad \& \quad \sigma_4 = \sqrt{10^2 + 10^2 + 10^2 + 10^2} \Rightarrow 
\]

25. A

\[
P(\text{Weight} > 800) = P\left( z > \frac{800 - \mu_4}{\sigma_4} \right) = P(z > 2) = 0.023
\]

26. A The number of distinguishable combinations of MISSISSIPPI is equal to \( \frac{11!}{4!2!4} \cdot \frac{11!}{4!2!4} = 34650 \). The number of distinguishable combinations of MISSISSIPPI where the P’s are in the first and last spots is equal to the number of distinguishable combinations of MISSISSII, which is \( \frac{9!}{4!4!} = 630 \) . And

\[
\frac{630}{34650} \approx 0.018 
\]

27. C An increase in the population variance causes the overlap between the hypothesized distribution and the true distribution to get larger. Thus the probability of committing a Type II error increases.

\[
n(\text{Analysis}) = n(\text{Linear Algebra}) \Rightarrow n(\text{Analysis} \cup \text{Linear Algebra}) = n(\text{Linear Algebra}) + \\
n(\text{Linear Algebra}) - n(\text{Analysis} \cap \text{Linear Algebra}) \Rightarrow 80 = 2 \cdot n(\text{Linear Algebra}) - 20 \\
\Rightarrow n(\text{Linear Algebra}) = 50
\]

28. C

\[
\text{n(Linear Algebra)} - \text{n(Analysis \cap Linear Algebra)} \Rightarrow 80 = 2 \cdot \text{n(Linear Algebra)} - 20 \\
\Rightarrow \text{n(Linear Algebra)} = 50
\]

29. D Systematic sampling occurs when you sample every nth person.

30. E The point \((15,26)\) is on the regression line. And since the data point \((3,6)\) has a residual of 4 the point \((3,2)\) is on the regression line because \( \text{Residual} = y - \hat{y} \). Now that you have two points on the line, you can find the equation of the line.

\[
a = 2 \quad \& \quad b = -4 \Rightarrow \frac{b}{a} \approx 0.840
\]