

Theta Topic Test - Functions
FAMAT State Convention 2004

For all questions, answer "E) NOTA" means none of the above answers is correct.
The figures in this test are not drawn to scale.

1) Given that $f(x-1) = 2x^2 + 3x - 10$, find $f(5) - f(3)$.

- A) 17 B) 38 C) 46 D) 54 E) NOTA

2) Solve the system of equations and find $x + y + z$.

$$3x + 2y - z = 6$$

$$y - x - 4z = 2$$

$$2z - 3y - 4x = 1$$

- A) -243 B) -105 C) -79 D) 243 E) NOTA

3) If $f(x) = 12x^5 + 3x^4 - 9x^3 - ax^2 + 20$, then which of the following could not be a root of $f(x)$?

- A) $-\frac{1}{2}$ B) 2 C) 5 D) 7 E) NOTA

4) If $f(x) = x^2 - x + 2$, then find the slope of the line containing the point $(m, f(m))$ and $(m + 3, f(m + 3))$.

- A) $m + 2$ B) $m + 3$ C) $2m + 1$ D) $2m + 2$ E) NOTA

5) A ball is thrown and follows the path $h(t) = -3t^2 + 16t + 6$ for h height after t seconds. What is the maximum height achieved by the ball?

- A) $\frac{8}{3}$ B) $\frac{82}{5}$ C) $\frac{82}{3}$ D) 41 E) NOTA

6) It is known that Q is a linear function of y on x . It is also known that $(2, 7)$ and $(-2, 5)$ satisfy Q . Find a value of x such that $Q(x) = 0$

- A) -12 B) -8 C) -7 D) 6 E) NOTA

7) The function $h(x)$ represents the greatest integer function, e.g. $h(x) = [x]$. Find the value of the following expression: $h(1.1) + h(\pi) - h(-2.1) + h(0)$.

- A) 4 B) 5 C) 6 D) 7 E) NOTA

8) For what values of x is $x^2 + 2x - 5 > 5$?

- A) $\{x : x < -\sqrt{11} - 1 \text{ or } \sqrt{11} - 1 < x\}$ C) $\{x : x > -\sqrt{11} - 1 \text{ and } \sqrt{11} - 1 > x\}$
B) $\{x : x < \sqrt{11} - 1 \text{ or } \sqrt{11} + 1 < x\}$ D) $\{ \}$ E) NOTA

- 9) Given that $f(x) = \frac{5}{2-3x}$, what is $f^{-1}(x)$ where $f(x)$ and $f^{-1}(x)$ are defined?
- A) $\frac{3x}{2x-5}$ B) $\frac{2x-5}{3x}$ C) $\frac{2x+5}{3x}$ D) $\frac{2x+5}{2x}$ E) NOTA
- 10) The graph of $h(x)$ has a range of $[-2,4]$ and a domain of $[0,1]$. What is the domain (D) and the range (R) of $f(x-3) + 5$
- A) D: $[5,6]$
R: $[-1,5]$ B) D: $[-3,-2]$
R: $[-1,5]$ C) D: $[3,4]$
R: $[3,9]$ D) D: $[-1,5]$
R: $[5,6]$ E) NOTA
- 11) From 10:00am to 4:00pm the temperature rose at a constant rate from 0°C to 25.7°C . What was the temperature at 3pm? (Note: Round to the nearest tenth of a degree.)
- A) 20.7 B) 21.4 C) 21.9 D) 22.4 E) NOTA
- 12) The space shuttle Endeavor at launch travels at a speed exponentially related to its time since launch, until it reaches orbit where it levels off. From launch until reaching orbit, its speed is governed by $S(t)$ such that t represents elapsed time in minutes since launch and S is measured in meters per second. It is known that $S(t) = e^{6t} - 1$. Assuming the velocity to safely penetrate the stratosphere must be between 19,000 and 24,000 meters per second, how large is the available time window to the shuttle (in seconds) after launch during which it can safely penetrate the stratosphere?
- A) 1.66 B) 2.34 C) 2.66 D) 2.99 E) NOTA
- 13) How many non-negative integer solutions are there to $3 - 16x > -100$?
- A) 0 B) 5 C) 7 D) ∞ E) NOTA
- 14) Solve for x to the nearest tenth: $e^{\ln(x+4)} + 3e^{\ln 3} = e^{\ln(5x-1)}$
- A) 2.5 B) 3.5 C) $\ln(3.5)$ D) $e^{3.5}$ E) NOTA
- 15) One of the roots of $x^2 + 4x + k = 0$ is ten more than the other. What is the value of k ?
- A) -42 B) -21 C) $\frac{-21}{2}$ D) $\frac{-21}{4}$ E) NOTA
- 16) Find the sum of all real x which satisfy $x^5 + 5x^4 - 19x^3 - 29x^2 + 42x = 0$.
- A) -7 B) -6 C) -5 D) -4 E) NOTA
- 17) What is the smallest possible value of $q(m) = 4m^2 - 10m + 3$ if m is restricted to integers.
- A) -3 B) -1 C) 0 D) 1 E) NOTA

26) A square of area A_s is inscribed in a circle of area A_c . Derive a functional relationship for A_s as a function of A_c .

- A) $A_s(A_c) = \frac{2\pi}{A_c}$ B) $A_s(A_c) = \frac{4\pi}{A_c}$ C) $A_s(A_c) = \frac{2A_c}{\pi}$ D) $A_s(A_c) = \frac{4A_c}{\pi}$ E) NOTA

27) Let R be a relation such that R is the following: $\{(1,1), (2, 4), (1, 3), (2, 5)\}$.
Is R^{-1} a function? If so, which is the correct set of ordered pairs for R^{-1} ?

- A) Yes, R^{-1} is a function. $(1,1), (2,4), (3,1), (5,2)$.
 B) Yes, R^{-1} is a function. $(1, 1), (2, 4), (1, 3), (2, 5)$.
 C) Yes, R^{-1} is a function. $(1,1), (4,2), (3,1), (5,2)$.
 D) No, R^{-1} is not a function.
 E) NOTA

28) If $M(x) = x^6 + 2x^5 - kx^2 + 2 - kx$ and it is known that $M(3) = 1157$. Find $P(-1)$.

- A) -1 B) 0 C) $\frac{1}{2}$ D) 1 E) NOTA

29) If $H(x) = \sqrt[4]{x^{32}}$ and the domain for H is all real numbers, what is the range of H ?

- A) $(-\infty, 0]$ B) $(-\infty, \infty)$ C) $[0, \infty)$ D) $\{\}$ E) NOTA

30) It is known that $F(x) = 2x^2 - x + 5$ and $G(x) = -x^2 + 15x + 1$. Furthermore, $H(x)$ is defined as:

$$H(x) = \begin{cases} F(x) - G(x) & \text{for } x > 0 \\ G(x) - F(x) & \text{for } x < 0 \\ \text{Max of } G(0) \text{ or } F(0) & \text{for } x = 0 \end{cases}$$

What is the maximum value of $H(x)$ on the interval $(-\infty, 5)$

- A) $\frac{-52}{3}$ B) -1 C) 0 D) 5 E) NOTA