## Theta Topic Test - Functions FAMAT State Convention 2004

For all questions, answer "E) NOTA" means none of the above answers is correct. The figures in this test are not drawn to scale.

1) Given that  $f(x-1) = 2x^2 + 3x - 10$ , find f(5) - f(3).

A) 17 B) 38 C) 46 D) 54 E) NOTA

2) Solve the system of equations and find x + y + z.

$$3x + 2y - z = 6$$
$$y - x - 4z = 2$$
$$2z - 3y - 4x = 1$$

A) -243 B) -105 C) -79 D) 243 E) NOTA

3) If  $f(x) = 12x^5 + 3x^4 - 9x^3 - ax^2 + 20$ , then which of the following could not be a root of f(x)?

A)  $\frac{-1}{2}$  B) 2 C) 5 D) 7 E) NOTA

4) If  $f(x) = x^2 - x + 2$ , then find the slope of the line containing the point (m, f(m)) and (m + 3, f(m + 3)).

A) m + 2 B) m + 3 C) 2m + 1 D) 2m + 2 E) NOTA

- 5) A ball is thrown and follows the path  $h(t) = -3t^2 + 16t + 6$  for *h* height after *t* seconds. What is the maximum height achieved by the ball?
  - A)  $\frac{8}{3}$  B)  $\frac{82}{5}$  C)  $\frac{82}{3}$  D) 41 E) NOTA
- 6) It is known that Q is a linear function of y on x. It is also known that (2, 7) and (-2, 5) satisfy Q. Find a value of x such that Q(x) = 0
  - A) -12 B) -8 C) -7 D) 6 E) NOTA
- 7) The function h(x) represents the greatest integer function, e.g. h(x) = [x]. Find the value of the following expression:  $h(1.1) + h(\pi) h(-2.1) + h(0)$ .
  - A) 4 B) 5 C) 6 D) 7 E) NOTA
- 8) For what values of x is  $x^2 + 2x 5 > 5$ ?

A) 
$$\left\{ x : x < -\sqrt{11} - 1 \text{ or } \sqrt{11} - 1 < x \right\}$$
  
B)  $\left\{ x : x < \sqrt{11} - 1 \text{ or } \sqrt{11} + 1 < x \right\}$   
D)  $\left\{ \right\}$   
E) NOTA

9) Given that  $f(x) = \frac{5}{2-3x}$ , what is  $f^{-1}(x)$  where f(x) and  $f^{-1}(x)$  are defined?

A) 
$$\frac{3x}{2x-5}$$
 B)  $\frac{2x-5}{3x}$  C)  $\frac{2x+5}{3x}$  D)  $\frac{2x+5}{2x}$  E) NOTA

- 10) The graph of h(x) has a range of [-2,4] and a domain of [0,1]. What is the domain (D) and the range (R) of f(x-3) + 5
  - A) D: [5,6]<br/>R: [-1,5]B) D: [-3,-2]<br/>R: [-1,5]C) D: [3,4]<br/>R: [3,9]D) D: [-1,5]<br/>R: [5,6]E) NOTA<br/>R) NOTA
- 11) From 10:00am to 4:00pm the temperature rose at a constant rate from 0°C to 25.7°C. What was the temperature at 3pm? (Note: Round to the nearest tenth of a degree.)
  - A) 20.7 B) 21.4 C) 21.9 D) 22.4 E) NOTA

12) The space shuttle Endeavor at launch travels at a speed exponentially related to its time since launch, until it reaches orbit where it levels off. From launch until reaching orbit, its speed is governed by S(t) such that *t* represents elapsed time in minutes since launch and S is measured in meters per second. It is known that  $S(t) = e^{6t} - 1$ . Assuming the velocity to safely penetrate the stratosphere must be between 19,000 and 24,000 meters per second, how large is the available time window to the shuttle (in seconds) after launch during which it can safely penetrate the stratosphere?

A) 1.66 B) 2.34 C) 2.66 D) 2.99 E) NOTA

13) How many non-negative integer solutions are there to 3 - 16x > -100?

A) 0 B) 5 C) 7 D)  $\infty$  E) NOTA

14) Solve for x to the nearest tenth:  $e^{\ln(x+4)} + 3e^{\ln 3} = e^{\ln(5x-1)}$ 

A) 2.5 B) 3.5 C) ln(3.5) D)  $e^{3.5}$  E) NOTA

15) One of the roots of  $x^2 + 4x + k = 0$  is ten more than the other. What is the value of k?

A) -42 B) -21 C) 
$$\frac{-21}{2}$$
 D)  $\frac{-21}{4}$  E) NOTA

16) Find the sum of all real x which satisfy  $x^5 + 5x^4 - 19x^3 - 29x^2 + 42x = 0$ .

A) -7 B) -6 C) -5 D) -4 E) NOTA

17) What is the smallest possible value of  $q(m) = 4m^2 - 10m + 3$  if m is restricted to integers.

A) -3 B) -1 C) 0 D) 1 E) NOTA

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18) Find u(w(v(-1))) given that u(x) = 2x - 1,  $v(x) = 2x^2$ , and  $w(x) = x^{-2}$ .

A) 
$$-\frac{1}{2}$$
 B) 0 C)  $\frac{1}{2}$  D) 2 E) NOTA

19) Find the sum of the coefficients of the binomial expansion of  $(2a^2 - 3b^4)^3$ .

A) -1 B) 0 C) 1 D) 2 E) NOTA

20) How many horizontal and vertical asymptotes (respectively) will the graph of f(x) have given

(A) 3,1 (b) 1,3 (c) 
$$\frac{x^3 - 3x^2 - 13x + 15}{x^3 + 3x^2 - 13x - 15}$$
?  
(c) 2, 1 (c) 1, 2 (c) NOTA

21) Given that the height of a rocket is governed by the equation  $h(t) = 15t - \frac{1}{3}t^2$ , find the time when the rocket is at its maximum height.

- A) 11.25 B) 16.5 C) 22.5 D) 45 E) NOTA
- 22) Suppose the graph of K lies completely and entirely in quadrant four. What quadrant does the graph of K<sup>-1</sup> lie in?
  - A) I B) II C) III D) IV E) NOTA
- 23) For a second degree polynomial, f(x), the sum of the roots is -6 and the product of the roots is -16. Find the sum of the reciprocals of the roots.
  - A) 1 B)  $\frac{1}{8}$  C)  $\frac{1}{4}$  D)  $\frac{3}{8}$  E) NOTA

24) Solve for x to the nearest hundredth.  $4.5 = \sqrt{x + 2 + \sqrt{x + 2 + \sqrt{x + 2 \dots x}}}$ .

A) 2.50 B) 12.00 C) 13.75 D) 14.00 E) NOTA

25) A company's end of year profit,  $p_n$ , is related to capital such that  $p_{n+1}=0.7p_{n-1}+0.6p_n$ . If a company made \$90 profit in 1999 and \$100 profit in the year 2000, what is its profit for 2004? (Round to the nearest cent)

A) \$143.80 B) \$172.38 C) \$204.09 D) 224.09 E) NOTA

26) A square of area  $A_s$  is inscribed in a circle of area  $A_c$ . Derive a functional relationship for  $A_s$  as a function of  $A_c$ .

A) 
$$A_s(A_c) = \frac{2\pi}{A_c}$$
 B)  $A_s(A_c) = \frac{4\pi}{A_c}$  C)  $A_s(A_c) = \frac{2A_c}{\pi}$  D)  $A_s(A_c) = \frac{4A_c}{\pi}$  E) NOTA

27) Let R be a relation such that R is the following: {(1,1), (2, 4), (1, 3), (2, 5)}. Is R<sup>-1</sup> a function? If so, which is the correct set of ordered pairs for R<sup>-1</sup>?

A) Yes, R<sup>-1</sup> is a function. (1,1), (2,4), (3,1), (5,2).
B) Yes, R<sup>-1</sup> is a function. (1, 1), (2, 4), (1, 3), (2, 5).
C) Yes, R<sup>-1</sup> is a function. (1,1), (4,2), (3,1), (5,2).
D) No, R<sup>-1</sup> is not a function.
E) NOTA

28) If  $M(x) = x^6 + 2x^5 - kx^2 + 2 - kx$  and it is known that M(3) = 1157. Find P(-1).

A) -1 B) 0 C) 
$$\frac{1}{2}$$
 D) 1 E) NOTA

29) If H(x) =  $\sqrt[4]{x^{32}}$  and the domain for H is all real numbers, what is the range of H?

A)(
$$-\infty, 0$$
] B) ( $-\infty, \infty$ ) C) [0,  $\infty$ ) D) {} E) NOTA

30) It is known that  $F(x) = 2x^2 - x + 5$  and  $G(x) = -x^2 + 15x + 1$ . Furthermore, H(x) is defined as:

$$H(x) = \begin{cases} F(x) - G(x) \text{ for } x > 0\\ G(x) - F(x) \text{ for } x < 0\\ Max \text{ of } G(0) \text{ or } F(0) \text{ for } x = 0 \end{cases}$$
 What is the maximum value of H(x) on the interval (-∞,5)

A) 
$$\frac{-52}{3}$$
 B) -1 C) 0 D) 5 E) NOTA