Solutions 2004 State Theta Geometry Topic Test

1. B The revolution of a rectangle about an axis forms a cylinder.

2. D Draw a pentagon and its diagonals and multiply the answer by 2.

3. D $7\frac{1}{2}^o$ can be constructed by bisecting a 60^o angle 3 times; $18\frac{3}{4}^o$ can be constructed by bisecting a 90^o angle 3 times to create 11.25^o and adding $7\frac{1}{2}^o$; $22\frac{1}{2}^o$ can be constructed using 15^o and $7\frac{1}{2}^o$ angles

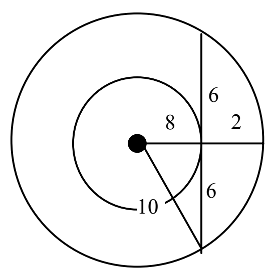
4. A The radii to the points where the inscribed circle intersects the triangle are perpendicular to the sides of the triangle.

5. B $\frac{1}{2} \bullet \frac{2r}{\sqrt{3}} \bullet r \bullet 6 = 96\sqrt{3}$; $r = 4\sqrt{3}$; $A = \pi\left(4\sqrt{3}\right)^2 = 48\pi$

6. E. The segment cannot form a triangle since the sum of two sides equals the third side.

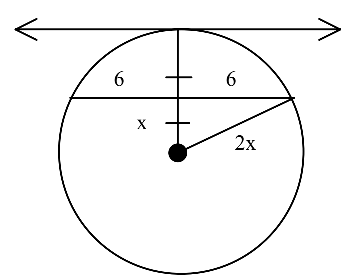
7. A Since P, Q, R, S, & T form inscribed angles and each angle intercepts arcs that form the circle, the sum is $\frac{1}{2} \bullet 360 = 180$.

8. A $\frac{1}{2}\begin{vmatrix} 4 & -6 & 1 \\ 1 & 5 & 1 \\ -3 & 2 & 1 \end{vmatrix} = 26.5$



9. A See figure to the right.

10. D $\frac{4}{3}\pi \bullet 3^3 = 36\pi$; $36\pi = \pi \bullet 5^2 \bullet H$; $H = \frac{36}{25}$; $6 + \frac{36}{25} = \frac{186}{25}$



11. C $(x+4)^2 + (y-5)^2 = 6^2$; $A = \pi \bullet 6^2 - \left(6\sqrt{2}\right)^2 = 36\pi - 72 \approx 41$

12. B $6^2 + x^2 = (2x)^2$; $x = 2\sqrt{3}$; $r = 2 \bullet 2\sqrt{3}$

13. B The bounded region forms a trapezoid; $A = \frac{1}{2}(4+6) \bullet 5$

14. A $\left(\frac{-4+2+5}{3}, \frac{-5+-2+4}{3}\right) = (1,-1)$

15. B KM = 7.5; LQ = 7.5; $A = \frac{1}{2}(7.5 + 22.5) \bullet 6$

16. B $x + z = 90$ & $y + z = 75$; $\therefore x - y = 15$

17. A NP = 12; PR = 6; PS = 5; PM = 10; RS = 10.5; $P = 5 + 6 + 10.5 = 21.5$

18. C $a^2 + 1^2 = 12^2$; $a = \sqrt{143} \approx 11.96$

19. B $1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 1 + \sqrt{2}$

20. D $\frac{w}{l} = \frac{l}{w+l}$; $\frac{3}{l} = \frac{l}{3+l}$; $l = \frac{3+3\sqrt{5}}{2}$

21. C $M = (5,10)$; $m_{\perp} = -\frac{3}{4}$; $y - 10 = -\frac{3}{4}(x-5)$; $3x + 4y = 55$

22. A A regular octahedron is 2 square pyramids with base and lateral edges of the same length. Find the volume of one pyramid and double it to get the volume of the entire octahedron. The altitude of one pyramid intersects a diagonal of the square base exactly at its midpoint, so a right triangle can be created to find the altitude of one pyramid. If a is the altitude of one pyramid, then $a^2 + \left(\frac{3\sqrt{2}}{2}\right)^2 = 3^2$ The altitude is $\frac{3}{\sqrt{2}}$. The volume of one pyramid is $\frac{1}{3} \bullet 3^2 \bullet \frac{3}{\sqrt{2}} = \frac{9}{\sqrt{2}}$.
Therefore, the volume of the octahedron is $2 \bullet \frac{9}{\sqrt{2}} = \frac{18}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}} = 9\sqrt{2}$.

23. A The reflexive property says that a segment is congruent to itself.

24. C $S = \frac{1}{2} \bullet (2 \bullet \pi \bullet 6) \bullet 10 + \pi \bullet 6^2 = 96\pi$

25. D $\sqrt{x^2 + 16} = 8 - x$; $x = 3$; $AB + BY = 5 + 13 = 18$

26. A Since a line with slope 1 forms a 45^o angle with the y-axis, a < 45.

27. A $(R \to T) \wedge (T \to S) \leftrightarrow (R \to S)$ by LS; $(U \to \sim S) \leftrightarrow (S \to \sim U)$ by LC;
$(R \to S) \wedge (S \to \sim U) \leftrightarrow (R \to \sim U)$ by LS; $(R \to \sim U) \wedge U \leftrightarrow \sim R$ by MT

28. B $\frac{5}{60} = \frac{x}{30}$; $x = 2.5$

29. A $D = \tan^{-1}\left(\frac{10 \tan 25^o}{16}\right) \approx 16$

30. D Two lines that are in the same plane never intersect.