LOGS AND EXPONENTS: THETA
Answers and Solutions
FAMAT State Convention 2004

Answer Key

1	C
2	B
3	B
4	A
5	C
6	D
7	C
8	A
9	E
10	B
11	B
12	D
13	D
14	C
15	A
16	A
17	C
18	A
19	C
20	E
21	C
22	D
23	D
24	A
25	C
26	B
27	D
28	B
29	C
30	B

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1) $\log_7(a-4)=2 \quad a-4=49 \quad a=53$ **C**

2) $2\cdot 4^{b-2}=32 \quad 4^{b-2}=16$
$b-2=\log_4 16=2 \quad b=4$ **B**

3) Units digits
$2^1=2 \quad 3^1=3$
$2^2=4 \quad 3^2=9$
$2^3=8 \quad 3^3=7$
$2^4=6 \quad 3^4=1 \Leftarrow$ 2004 divisible
$2^5=2 \quad 3^5=3$ by 4
Units digit for powers of 5 is always 5
$6+1-5=2$ **B**

4) $16^{\sqrt{x}}=\left(4^2\right)^{\sqrt{x}}=4^{2\sqrt{x}}$ IV. only **A**

5) $\begin{vmatrix} \log a & -1 \\ \log(a-1) & 2 \end{vmatrix}=2\log a+\log(a-1)=$
$\log a-\log\left(\frac{1}{2}\right)=\log a+\log 2$
$2\log a+\log(a-1)=\log a+\log 2$
$a(a-1)=2 \quad a^2-a-2=0$
$a=-1,2$ $\quad a$ must be positive;
sum = 2 **C**

6) $9^x-2\cdot 3^{x+1}-7=0$
$3^{2x}-6\cdot 3^x-7=0$ Let $y=3^x$
$y^2-6y-7=0 \quad y=7,-1$
y cannot be negative.
$7=3^x \quad x=\log_3 7=1.7\underline{7}12...$ **D**

7) $\sqrt{182+x}=x \quad x^2-x-182=0$
$x=14,-13$ Since x cannot be negative, it must equal 14. **C**

8) $6531=a+b\ln 1 \quad a=6531$
$8634=6531+b\ln 2 \quad b=3304$
$y=6531+3034\ln 3=9864$ **A**

9) $\sum_{i=1}^{4}\log_2 i=\log_2(1\cdot 2\cdot 3\cdot 4)=\log_2 24$ **E**

10) $2a-b+\frac{3c}{2}=2\log x-\log y+\frac{3}{2}\log z=$
$\log\left(\frac{x^2\cdot\sqrt{z^3}}{y}\right)=2\log\left(\frac{x\cdot\sqrt[4]{z^3}}{\sqrt{y}}\right)$ **B**

11) $\log_4 2^{100}=\frac{1}{2}\log_2 2^{100}=\frac{1}{2}\cdot 100=50$
Answer: 2^{50} **B**

12) $\log\left(\frac{A\cdot B^2}{C^3}\right)=\log A+2\log B-3\log C$
$\frac{5}{2}+2\left(\frac{9}{2}\right)-3\left(-\frac{3}{2}\right)=16$ **D**

13) The graphs will intersect once in the second quadrant and twice in the first quadrant. **D**

14) I. and IV. have two values of x for one value of y, therefore they do not have inverses. II. and III. are strictly increasing functions, therefore they do have inveses. **C**

15) $\left(-1024^{\frac{1}{2}}x^2\right)^{\frac{-4}{5}}=\left(-1024^{\frac{1}{2}}\right)^{\frac{-4}{5}}\left(x^2\right)^{\frac{-4}{5}}=$
$\left(-1024^{\frac{-2}{5}}\right)x^{\frac{-8}{5}}=\frac{1}{16x^{8/5}}$ **A**

16) $5=\log_2(x!+8) \quad x!+8=32 \quad x!=24$
$x=4$ **A**

17) $y=Pe^{rt} \quad P=1525.00$
$1642.13=1525.00e^{r(1)}$
$r=\ln\left(\frac{1642.13}{1525.00}\right)=0.074$
$3050=1525e^{0.074t}$
$t=\frac{\ln 2}{0.074}=9.4\,years$ **C**

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18) $mantissa \times 10^{characteristic}$
$A = 3$, $B = 6.63$
$4^{(3x)} = 3^{(6.63x+1)}$
$(3x)\log_{10} 4 = (6.63x+1)\log_{10} 3$
$6.63x + 1 = 3.785579x$
$x = -0.35$ **A**

19) $a^{12}b^{12}$ is the 7th term.
${}_{10}C_6\left(\frac{2}{3}\right)^4\left(\frac{1}{2}\right)^6 = \frac{35}{54}$ **C**

20) $x^2 - 2 > 0 \qquad x^2 > 2$
$x > \sqrt{2} \cup x < -\sqrt{2}$ **E**

21) $\prod_{i=3}^{n} \log_i (i+1)$
$= (\log_3 4)(\log_4 5)\cdots(\log_n (n+1))$
$= (\log_4 4)(\log_5 5)\cdots(\log_3 (n+1))$
All the terms cancel except for
$(\log_3 (n+1))$ **C**

22) $\left[(-i)^{-1} + \left(\frac{1}{i^3}\right)^5\right]^4 = \left[i + i^5\right]^4 =$
$[i+i]^4 = (2i)^4 = 16i^4 = 16$ **D**

23) $\log_{x^2} y = \frac{1}{9} \qquad \log_x y = \frac{2}{9}$
$\log_y x = \frac{1}{6}\log_{\sqrt{y}} x^3 = \frac{9}{2}$
$\log_{\sqrt{y}} x^3 = \frac{9}{2} \cdot 6 = 27$ **D**

24) $50^a 125^b 8^c = 1000$
Since $1000 = 125 \cdot 8$:
$a = 0, b = 1, c = 1 \qquad a + b + c = 2$ **A**

25) $\log_y (y+12) = 2 \quad y^2 - y - 12 = 0$
$y = 4, -3$ $\quad y$ cannot be negative
sum = 4 **C**

26) $\sum_{z=1}^{\infty} \frac{1}{2^z} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$
This is a geometric series with $\frac{1}{2}$ as the first term and a ratio of $\frac{1}{2}$.
Sum = $\dfrac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$ **B**

27) $\log_7(\log_5(\log_2 x)) = 0$
$\log_5(\log_2 x) = 1 \quad \log_2 x = 5 \quad x = 32$ **D**

28) I. False: $r = \log_a x$
II. True: $y = (a^r)^2 = x^2$
III. False: $z = 2a^r = 2\sqrt{y} \qquad 4y = z^2$
IV. True: $\frac{z}{2} = a^r \quad \log_a\left(\frac{z}{2}\right) = r$
2 of these are true **B**

29) $146\#1 = 47\#2 = 14\#3 = 3\#4$
$3^4 = 81$ **C**

30) $2004^{2004} = x \quad \log_{10}(2004^{2004}) = \log_{10} x$
$\log_{10} x = 6617.0030...$
The number of digits is always the smallest integer greater than the base-10 logarithm of the number. 6618 digits **B**