QUADRILATERALS TOPIC TEST

FAMAT State Convention 2004

For all questions, choice E is NOTA, meaning “None Of These Answers.”

Diagrams are not necessarily drawn to scale!

1. A square has an apothem of 12. What is the area of the square?
   A) 72    B) 144    C) 288    D) 576    E) NOTA

2. A quadrilateral has sides which measure \( \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \) and \( \frac{7}{8} \). What is the perimeter of the quadrilateral?
   A) \( \frac{7}{16} \)    B) \( \frac{19}{22} \)    C) \( \frac{15}{4} \)    D) \( \frac{391}{120} \)    E) NOTA

3. What is the sum of the exterior angles of a convex quadrilateral?
   A) 180°    B) 360°    C) 720°    D) 1080°    E) NOTA

4. The midline of an isosceles trapezoid measures 12, and is a distance of 4 from the smaller base. What is the area of the trapezoid?
   A) 48    B) 64    C) 76    D) 96    E) NOTA

5. Which statement is not necessarily true of a parallelogram?
   A) Its opposite sides are congruent.
   B) Its diagonals bisect each other.
   C) Its opposite angles are supplementary.
   D) Its opposite sides are parallel.
   E) NOTA

6. A square with an area of 1600 is being shrunk, but in such a way that it always remains a square. First, it is shrunk so that its perimeter is reduced to 25% of the original perimeter. The resultant square is then shrunk so that its perimeter is reduced by 10%. What is the area of the square after all this reduction?
   A) 36    B) 81    C) 100    D) 360    E) NOTA

7. Quadrilateral PSAT is inscribed in the circle shown below. If \( m\angle ASP = 94° \) and \( m\angle SAT = 80° \), then what is \( m\angle PTA \)?
   A) 86°    B) 87°    C) 90°    D) 100°    E) NOTA

8. A quadrilateral’s sides are segments of the lines given by the equations \( x = 0, \ y = 2, \ y = -4, \) and \( y = -2x + 4 \). Its vertices are the points of intersection of the lines. What is the area of the quadrilateral?
   (NOTE: The point (0, 4) is not a vertex of the quadrilateral.)
   A) 15    B) 18    C) 21    D) 24    E) NOTA

9. What is the product of the lengths of the diagonals of a rectangle which has dimensions of 10 and 20?
   A) 200    B) 225    C) 300    D) 500    E) NOTA
10. In rectangle \(MNPQ\), diagonals \(MP\) and \(NQ\) intersect at point \(X\). If \(MQ = 8\) and \(MP = 10\), then what is the value of \((NX + PQ)\)?

A) 11    B) 13    C) 16    D) 18    E) NOTA

11. A circle with a diameter of 3 is inside of a rectangle with dimensions 8 and 10 as shown below. What is the area of the shaded region, rounded to the nearest integer? (Use \(\pi = 3.14\))

A) 52    B) 61    C) 71    D) 73    E) NOTA

12. A length of fencing is to be used to enclose a quadrilateral-shaped area. Most specifically, what quadrilateral must be made to ensure that the largest area possible is enclosed?

A) Isosceles Trapezoid    B) Rectangle    C) Rhombus    D) Square    E) NOTA

13. Parallelogram \(MATH\) has an area of 200. If \(m\angle TAM = 50^\circ\) and \(AM = 10\), then what is the length of \(AT\), rounded to the nearest tenth?

A) 20.0    B) 22.4    C) 26.1    D) 31.1    E) NOTA

14. The product of the lengths of the diagonals of a square is 12. What is the area of the square?

A) 6    B) 12    C) 24    D) 36    E) NOTA

15. A square is inscribed inside of a square, which is inscribed inside of a third square, as shown below. \(CB \cong CA\), \(EC \cong ED\), and \(AB = 20\). What is the area of the shaded region?

A) 12.5    B) 12.5\(\sqrt{2}\)    C) 25    D) 50    E) NOTA

16. In kite \(WIDE\), \(ID \cong ED\) and \(EI\) intersects \(WD\) at point \(N\). If \(m\angle WIN = 75^\circ\) and \(m\angle NWI = 15^\circ\), then what is \(m\angle EDN\)?

A) 15\(^\circ\)    B) 30\(^\circ\)    C) 45\(^\circ\)    D) 60\(^\circ\)    E) NOTA

17. One face of a right rectangular prism has an area of 6. Another face of the same prism has an area of 16. The volume of the prism is 48. What is the prism’s surface area?

A) 45    B) 70    C) 92    D) 144    E) NOTA

18. A rectangle \(R\) has a perimeter of 8. A rectangle \(Q\) has three times the length and four times the width of \(R\). The perimeter of \(Q\) is 30. What is the sum of the lengths of \(R\) and \(Q\)?

A) 4    B) 5    C) 8    D) 12    E) NOTA
19. Which of the following three statements must be true?
   I. All rectangles have perpendicular diagonals.
   II. All trapezoids are concave.
   III. A diagonal in any rhombus bisects opposite angles of the rhombus.
   A) I only       B) II only       C) III only
   D) I and III only   E) NOTA

20. The vertices of parallelogram CLAP lie on rectangle ABCD as shown below.
   CL = 17, CB = 15, and LP || AD. What is the area of parallelogram CLAP?
   A) 60       B) 120
   C) 136       D) 240
   E) NOTA

21. The area of kite ABCD (shown below) is 252.
   AC = 24 and BC = 13. Diagonals AC and BD intersect at point E.
   Find the value of (DE + DC).
   A) 16       B) 8 + 4√3
   C) 36       D) 41
   E) NOTA

22. In the isosceles trapezoid ABCD, BC = 6, AB = 8, and m∠BCD = 30°. What is the measure of diagonal AC, rounded to the nearest integer?
   A) 9       B) 14
   C) 15       D) 18
   E) NOTA

23. How many distinct squares with positive perimeters exist which satisfy the condition that the area of the square must equal its perimeter?
   A) 0       B) 1       C) 2       D) ∞       E) NOTA

24. A rhombus with a diagonal of 12 has an area of 21. What is the perimeter of the rhombus?
   A) 14       B) 4√21       C) 25       D) 24√2       E) NOTA

25. A quadrilateral is inscribed in a circle as shown below. The quadrilateral’s side measurements are given in the diagram. What is the area of the quadrilateral, rounded to the nearest tenth?
   A) 104.5       B) 105.3
   C) 109.3       D) 110.0
   E) NOTA

26. Rectangle ABCD has an area of 45. Rectangle WXYZ has an area of 80. □ABCD ~ □WXYZ. If the perimeter of rectangle ABCD is 28, then what is the perimeter of rectangle WXYZ?
   A) \(\frac{112}{3}\)       B) \(\frac{448}{9}\)
   C) \(\frac{2102}{27}\)       D) \(\frac{7168}{81}\)
   E) NOTA
27. Trapezoid $QDEN$ (shown below) has an area of 512. If $QD = 2x + 8$, $QN = 3x - 5$, and $EN = 6x$, then what is the product $((QN)(QD))$?

A) 180  B) 260  C) 352  D) 456  E) NOTA

28. In the quadrilateral $HARD$ shown below, $m\angle HDR = 115^\circ$ and $m\angle ARD = 120^\circ$. The following measurements are known to be accurate to the nearest tenth:

$HA = 66.0$, $RD = 30.0$, $AR = 25.0$

What is the area of the quadrilateral $HARD$, rounded to the nearest integer?

A) 1158  B) 1200  C) 1530  D) 1645  E) NOTA

29. How many distinct parallelograms exist with positive integral side lengths and perimeters less than 11?

A) 6  B) 12  C) 21  D) 25  E) NOTA

30. There exist two isosceles trapezoids that have the same perimeter ($P$) and the same height ($H$). The first trapezoid has a base angle measuring $30^\circ$ and an area $A_1$. The second trapezoid has a base angle measuring $45^\circ$ and an area $A_2$.

If the ratio $\frac{A_1}{A_2}$ can be written in the form $\frac{aP - bH}{cP - dH}$, then find the value of $\frac{ab}{cd}$ and rationalize the result.

A) $2(2 - \sqrt{3})(1 - \sqrt{2})$  B) $2(2 + \sqrt{3})(1 - \sqrt{2})$

C) $\sqrt{2}$  D) $\frac{1 - \sqrt{2}}{2}$  E) NOTA