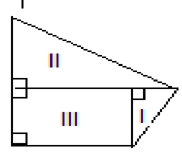
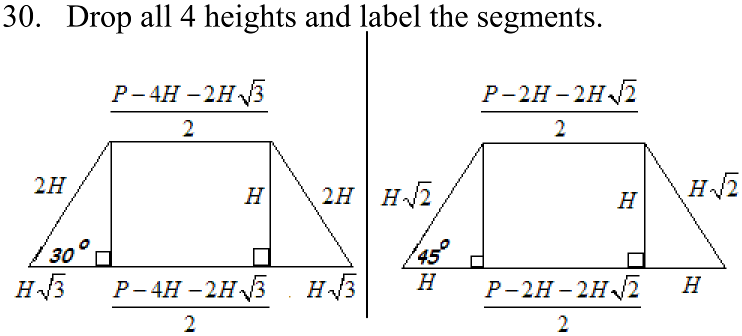
2004 STATE QUADRILATERALS TOPIC TEST
-------SOLUTIONS-------

1. Each side is 24. Its area is 576, labeled with a **D**.
2. The sum of the sides is 391/120. My vote is for **D**.
3. In any convex polygon, the sum is 360^o, choice **B.**
4. The distance to the midline from a base is half the height. So the area is (8)(12), or 96. Choice **D**.
5. Opposite angles of a parallelogram are congruent, which means that choice **C** is a filthy lie.
6. The perimeter of 160 is reduced to 40, then to 36. Each side is 9 and the area is 81, choice **B**.
7. Opposite angles of a quadrilateral inscribed in a circle are supplementary. Choice **A**, 86^o.
8. The quadrilateral can be subdivided into a rectangle with area 6 and right triangle with area 9. The sum is 15, choice **A**.
9. Each $\cong$ diagonal $=\sqrt{500}$. $(\sqrt{500})^2 = 500$; it's **D**.
10. PQ=6. The diagonals bisect each other and are congruent, so NX = 5. The sum is 11, choice **A**.
11. The area is $80-1.5^2\pi \approx 73\ldots$ choice **D** (for **d**efinition of **d**iameter!)
12. The quadrilateral must have all right angles to avoid area loss by law of cosines. Let the rectangle have length L and width (.5P-L), where P is the perimeter. The maximum of A = L(.5P-L) is at L=P/4. If L=P/4, then W = P/4. The shape is a square. Choice **D**.
13. Altitude to $\overline{AM} = 20$; $AT = \dfrac{20}{\sin 50°} \approx 26.1$. Vote **C**!
14. Each diagonal is $\sqrt{12}$, and forms a 45-45-90 triangle. Each side is $\sqrt{6}$. The area is 6, choice **A**.
15. Symmetry reveals that it is a 45-45-90 triangle with legs of $5\sqrt{2}$. The area is 25, as you can **C**.
16. Not enough information – make the **E**-z choice.
17. Let the edges of the prism be x, y, and z. (xyz) = 48; (xy) = 6; (yz) = 16. Substitution reveals that x = 3, y = 6, and z = 8. The surface area is 2(xy + yz + xz), or 92. Feels like a **C**.
18. Solving the system L+W = 4; 3L + 4W = 15, we find that L = 1. L + 3L = 4, earning an **A**.
19. Statement III alone is true (I is false, and II is just plain ridiculous!) Choice **C** is the way to go.
20. PL = 15, and PC = 8. The sum of the areas of congruent triangles LAP & PCL = 120, choice **B**.
21. The diagonals are perpendicular. AE is 12, so BE is 5. The area of each small triangle is 30, so the area of each large triangle is 96. Hence, DE is 16 and then DC = 20. 16 + 20 = 36, choice **C**.
22. Law of cosines: $AC = \sqrt{6^2+8^2-2(6)(8)\cos 150°}$, which rounds to 14. Let it **B**.
23. $x^2 = 4x$ has only 1 positive solution (4), choice **B**.
24. Each of the 4 congruent right triangles formed by the diagonals has area = 5.25, and one leg = 6. Other leg = 7/4. Hypotenuse, which is the side of the rhombus = 25/4. (25/4)(4) = 25, choice **C**.
25. $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$,by Brahmagupta's Generalization. $s = 21$, so $A \approx 104.5 =$ **A**.
26. The perimeter of ABCD is 28. The perimeter ratio is the square root of the area ratio. Solving $\sqrt{\dfrac{45}{80}} = \dfrac{28}{x}$ gives $x = \dfrac{112\sqrt{5}}{3\sqrt{5}} = \dfrac{112}{3}$, choice **A**.
27. The mean of the bases is 4x + 4. Solving the quadratic $(3x-5)(4x+4) = 512$ yields one real solution, $x = 7$. $QD = 22$, $QN = 16$, and the product is 352. Wipe your brow and bubble **C**.
28. Draw the altitudes shown. Find that I is a 30-60-90 triangle with short leg (adjacent to II) of 15 and long leg (adjacent to III) of $15\sqrt{3} \approx 26$. II is a 45-45-90 triangle with legs of (66 - 26) = 40. III is a rectangle with dimensions 26 and 25. The areas of I, II, and III respectively are 195, 800, and 650. The sum is 1645. Quite **D**-manding!



29. Parallelograms are distinguished both by their side lengths and by their angles. Take rhombi with sides of 2. One such rhombus exists with an 80^o angle, one with a 79^o angle, one with a 64^o angle, etc. The answer is ∞, which is not given -> **E**.
30. Drop all 4 heights and label the segments.



Since the areas of both trapezoids are calculated by the same formula, they have the factors (1/2) and H in common. These will cancel, so the area ratio is the same as the ratio of the sums of the bases. In the 30^o trapezoid, the sum of the bases is $P-4H$. In the 45^o trapezoid, the sum is $P-(2\sqrt{2})H$. Many equivalent fractions may be made, but the ratio $\dfrac{ab}{cd}$ that we are looking for is always the same.

$\dfrac{ab}{cd} = \dfrac{(1)(4)}{(1)(2\sqrt{2})} = \sqrt{2}$, which is choice **C**.