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THETA BOWL
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1) A = 6; B = 3; C = 3

$$\frac{6 \cdot 3}{3} = \mathbf{6}$$

2) Points of intersection.

$x^2 y^2 = 8$

$x^2 + (\frac{1}{2}x + 1)^2 = 8$ (substitution)

$x^2 + \frac{1}{4}x^2 + x + 1 = 8$

$\frac{5}{4}x^2 + x - 7 = 0$

$5x^2 + 4x - 28 = 0$

$(x - 2)(5x + 14) = 0$

$x = 2 \quad x = \frac{-14}{5}$

$y = \frac{1}{2}(2) + 1 = 2 \qquad y = \frac{1}{2}(\frac{-14}{5}) + 1 = \frac{-2}{5}$

A $= 2 + (\frac{-14}{5}) = \frac{-4}{5}$; **B** $= 2 + (\frac{-2}{5}) = \frac{8}{5}$

C = 0 (the eccentricity of any circle)

D = center of $x^2 + 4y^2 + 6x - 8y + 9 = 0$

$x^2 + 6x + 9 + 4(y^2 - 2y + 1) = -9 + 9 + 4$

$(x + 3)^2 + 4(y - 1)^2 = 4$

center (-3, 1)

-3 + 1 = -2

ABD + C = $(\frac{-4}{5})(\frac{8}{5})(-2) + 0 = \mathbf{\frac{64}{25}}$ **or 2.56**

3) $x^3 + y^3 = 35$ and $x + y = 5$. In factored form, $(x + y)(x^2 - xy + y^2) = 35$. Substituting, $5(x^2 - xy + y^2) = 35$ → $(x^2 - xy + y^2) = 7$. Substituting $5 - y$ for x, you get $(5 - y)^2 - (5 - y)(y) + y^2 = 7$. Simplify to $y^2 - 5y + 6 = 0$. Solve to get y = 3 or y = 2. When y = 3, x = 2 and when y = 2, x = 3. In either case, the container with the largest Volume is 3^3 or **27**.

4) A – 6; B – 2; C – 1; D – 9; E – 10; F – 5

6 ● 5 – (2 + 1) ● 10 + 9 = **9**

5) $f(x) = \frac{(x+1)(x-3)}{(x+5)(x-3)}$

A : $\left(3, \frac{1}{2}\right)$ → $3 + \frac{1}{2} = \frac{7}{2}$; **B :** x = -5; **C :** $y = \frac{1}{1} = 1$

$ABC = (\frac{7}{2})(-5)(1) = \mathbf{\frac{-35}{2}}$

6) **A :** 11! = 39916800; **B :** 10! = 3628800;

C : 1 ● 10 ● 2 ● 8 ● 7 ● 6 ● 5 ● 4 ● 3 ● 2 ● 1 = 806400;

D : $\frac{11!}{2!3!2!2!} = 831600$

$$\frac{A}{B + C + D} \approx \mathbf{7.58}$$

7) 79 + 87 + 68 –30 – 28 – 23 + 12 = 165

195 – 165 = **30**

8) **A :** $231_6 = (2 \bullet 6^2) + (3 \bullet 6^1) + (1 \bullet 6^0)$

= 72 + 18 + 1 = 91

91 ÷ 64 → remainder 27 ÷ 8 → remainder 3

--→ 133_8

B : 2004 ÷ 625 → remainder 129 ÷ 125 → remainder 4 ÷ 25 → r. 4 ÷ 5 → r. 4 ÷ 1 --→ 31004_5

C : $t_1 = 2(1) - 3 = -1$; $t_{20} = 2(20) - 3 = 37$

$S_{20} = \frac{20}{10}(-1 + 37) = 360$

A + B + C = **31497**

9) **A** $= \frac{0.38}{(0.0004)(24^2)} \approx 1.65$

B will reach a maximum when $1.101e^{-0.051t}$ approaches zero. ∴ the max. pop. Is 11.14

A – B = **-9.49**

10) **A :** 1 – 0.16 = 0.84; **B :** 1 – 0.21 = 0.79;

C : (0.84)(0.79) = 0.6636; **D :** 1 – 0.6636 = 0.3364;

E : (0.16)(0.21) = 0.0336

$$\frac{0.84 + 0.79 + 0.0336}{0.6636 + 0.3364} = \mathbf{1.6636}$$

11) $A = \frac{1}{2}d_1 d_2$ → $264 = \frac{1}{2}(12)(d_2)$ → $d_2 = 44$

diagonals are 12' and 44 ' long.

$s^2 = 6^2 + 22^2 = 520$ → $s \approx 22.8$

4(22.8) = 91.2 → **92**

12) **M :** I = PRT

I = (5000)(.06)(5)

= 1500

A = P + I

= 5000 + 1500

= \$6500

S : $A = P(1 + \frac{r}{n})^{nt}$

$A = 5000(1.01375)^{20}$

\$6570.33

D : $A = Pe^{rt}$

$= (5000)e^{0.25}$

= \$6420.13

\$6570.33; \$6500; \$6420.13

\$6570.33 - \$6420.13 = **\$150.20**

13) **A :** (345)(0.8) = 276; (276)(2) = 552

B: $S_n = t_1 \bullet \frac{1 - r^n}{1 - r}$ → $S_5 = 552 \bullet \frac{1 - (.8)^5}{1 - (.8)} = 1856$

C: $S = \frac{t_1}{1 - r}$ → $S = \frac{552}{1 - (.8)} + 345 = 3105$

A + B + C = 552 + 1856 + 3105 = **5513**

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14) **A :** $2^x = 1024 \rightarrow x = 10$

B : $x^{-3} = 125 \rightarrow x = \frac{1}{5}$

C : $x^{\frac{3}{4}} = 125 \rightarrow x = 625$

D : $4^{\frac{-1}{2}} = x \rightarrow \quad x = \frac{1}{2}$

ABCD = $10 \bullet \frac{1}{5} \bullet 625 \bullet \frac{1}{2}$ = **625**

15) **A :** $\frac{6}{\sqrt[5]{64}} = \frac{6}{2\sqrt[5]{2}} = \frac{3}{\sqrt[5]{2}} \bullet \frac{\sqrt[5]{2^4}}{\sqrt[5]{2^4}} = \frac{3\sqrt[5]{16}}{2} \rightarrow A = 2$

B) $\sqrt[4]{2^3} \div \sqrt[6]{2^4} \bullet \sqrt[12]{2^7}$

$2^{\frac{3}{4}} \div 2^{\frac{4}{6}} \bullet 2^{\frac{7}{12}}$

$2^{\frac{9}{12}-\frac{8}{12}+\frac{7}{12}} = 2^{\frac{8}{12}} = 2^{\frac{2}{3}}$

$\rightarrow B = \frac{2}{3}$

AB = $2 \bullet \frac{2}{3} = \mathbf{\frac{4}{3}}$