2004 National Mu Alpha Theta Convention
Alpha Division–Number Theory Topic Test

1. What is the largest prime divisor of 70?

A. 2 B. 3 C. 7 D. 10 E. NOTA

2. What is the smallest positive integer consisting solely of 1's which is divisible by three?

A. 1 B. 11 C. 111 D. 1111 E. NOTA

3. What is the smallest positive integer with three different prime factors?

A. 8 B. 30 C. 60 D. 72 E. NOTA

4. Zero is not a positive number. If it were considered to be positive, which of the following statements would no longer be considered true:
I. A negative number times a positive number is always negative
II. A positive number plus a positive number is always positive
III. A positive number times a positive number is always positive
IV. A negative number times a negative number is always positive

A. I only B. II only C. I and III D. IV only E. NOTA

5. If a is even and both $\sqrt{a}$ and $\sqrt[3]{a}$ are integers, then what is the largest integer which must divide a?

A. 64 B. 128 C. 256 D. 512 E. NOTA

6. What is the smallest integer n such that the sum of the first n positive integers is divisible by 13?

A. 11 B. 12 C. 13 D. 14 E. NOTA

7. How many positive even two-digit numbers are multiples of 7?

A. 6 B. 7 C. 8 D. 9 E. NOTA

8. How many primes are 1 less than a perfect square?

A. 0 B. 1 C. 3 D. infinitely many E. NOTA

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9. Ninety-nine balloons are released in the cafeteria and float to the ceiling. Jimmy pops n of them. Claire then enters and pops exactly 1/3 of those remaining. The principal then takes exactly 1/4 of the inflated balloons remaining after all that popping. If the principal takes at least one balloon, how many possible values are there of n? (Assume one cannot pop or take a part of a balloon.)

A. 2 B. 8 C. 16 D. 99 E. NOTA

10. When k is divided by 7, the result has a remainder of 3. When j is divided by 7, the result has remainder 4. What is the remainder when $k + j$ is divided by 7?

A. 0 B. 4 C. 6 D. Cannot be determined. E. NOTA

11. For how many positive integers n less than 9 is $n! + 1$ divisible by n?

A. 0 B. 1 C. 3 D. 8 E. NOTA

12. Which of the following is the smallest: 2^{500}, 3^{400}, 4^{300}, 5^{200}?

A. 2^{500} B. 3^{400} C. 4^{300} D. 5^{200} E. NOTA

13. What is the greatest common divisor of 3618 and 938?

A. 2 B. 18 C. 67 D. 134 E. NOTA

14. I write the integers from 2 to 1000 on a piece of paper. I then circle all the primes. I then circle all the previously uncircled numbers which are multiples of 2, 3, 5, or 7. What is the smallest uncircled number remaining?

A. 11 B. 121 C. 143 D. 169 E. NOTA

15. Which of the following is the list of possible units digit of a perfect square that ends with 4 identical digits?

A. 0 only B. 0 or 4 only C. 0, 1, or 4 only D. 0, 1, 4, or 6 only E. NOTA

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16. We say $x|y$ if x divides y evenly. If $m|n$ and $n|m$, which of the following can we conclude?

A. $m > n$ B. $m = n$ C. $m = 1$ D. $mn \not| m$ E. NOTA

17. Two numbers are called *relatively prime* if their greatest common divisor is 1. How many numbers less than 120 are relatively prime to 120?

A. 32 B. 36 C. 40 D. 48 E. NOTA

18. Find the largest integer that evenly divides $n^5 - 5n^3 + 4n$ for all integers n.

A. 24 B. 60 C. 120 D. 240 E. NOTA

19. Let n be a randomly chosen two-digit positive integer. What is the probability that the fraction $6/n$ is reducible?

A. 1/2 B. 2/3 C. 4/5 D. 5/6 E. NOTA

20. Find the sum of all positive integers x for which there exists a y such that $x^2 - y^2 = 35$

A. 6 B. 18 C. 24 D. 48 E. NOTA

21. For how many primes p is $2^p + p^2$ also prime?

A. 0 B. 1 C. 3 D. Infinitely many E. NOTA

22. For how many pairs of positive integers (a, b) is $\gcd(a, b) \times \text{lcm}(a, b) = ab$, where $\gcd(x, y)$ is the greatest common divisor of x and y and $\text{lcm}(x, y)$ is the least common multiple of x and y?

A. None B. 25 C. 144 D. All pairs E. NOTA

23. If m is divisible by 420 and n is divisible by 294, then what is the largest integer which divides $m + n$ for all values of m and n?

A. 6 B. 7 C. 14 D. 42 E. NOTA

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24. How many solutions in positive integers (x, y) are there to the equation $8x + 5y = 107$?

A. 2 B. 4 C. 6 D. 7 E. NOTA

25. A, B, and C are nonnegative integers less than 10. $100A + 10B + C$ is divisible by 5. $10A + C$ is divisible by 3. $10B + C$ is divisible by 4. $10A + B$ is divisible by 9. Find $A + B + C$.

A. 3 B. 9 C. 12 D. Cannot be determined E. NOTA

26. How many positive numbers less than 729 have a base 3 representation with no 2's?

A. 63 B. 81 C. 100 D. 144 E. NOTA

27. For how many positive integers k is 120 the least common multiple of 40 and k?

A. 2 B. 6 C. 8 D. 12 E. NOTA

28. The number j^2, where j is an integer, leaves a remainder of k when divided by n $(k < n)$. How many possible pairs of values (k, n) are there with $1 < n < 10$.

A. 22 B. 23 C. 24 D. 26 E. NOTA

29. We define the set of **cycles** of an integer to be all the integers that are formed by successively moving the units digit to the front and moving all the other digits over one place to the right. For example, the set of cycles of 4275 is $\{5427, 7542, 2754, 4275\}$ and the set of cycles of 51904 is $\{45190, 04519, 90451, 19045, 51904\}$. (Note the treatment of the 0 - it doesn't disappear as we form our cycles!)

How many positive 10-digit integers n (i.e. $10^{10} > n \geq 10^9$) have the property that all members of the set of cycles of n are divisible by 11111?

A. 900009 B. 810009 C. 1110000 D. 0 E. NOTA

30. Given $f(1) = 1$, $f(2n) = f(n)$ and $f(2n + 1) = f(2n) + 1$ for all integers n, for how many values of n is $f(n) = 10$ for $n < 2003$?

A. 2 B. 5 C. 6 D. 9 E. NOTA