2004 National Mu Alpha Theta Convention
Mu Division–Number Theory Topic Test

1. What is the least common multiple of 72 and 30?

A. 180 B. 360 C. 540 D. 720 E. NOTA

2. What is the smallest positive integer with three different prime factors?

A. 8 B. 30 C. 60 D. 72 E. NOTA

3. Zero is not a positive number. If it were considered to be positive, which of the following statements would no longer be considered true:
I. A negative number times a positive number is always negative
II. A positive number plus a positive number is always positive
III. A positive number times a positive number is always positive
IV. A negative number times a negative number is always positive

A. I only B. II only C. I and III D. IV only E. NOTA

4. Find the sum of all values of A such that the three digit number $6A4$ is divisible by 3.

A. 7 B. 10 C. 12 D. 15 E. NOTA

5. What is the largest prime factor of 100! ?

A. 91 B. 97 C. 10001 D. Cannot be determined E. NOTA

6. What is the smallest integer n such that the sum of the first n positive integers is divisible by 13?

A. 11 B. 12 C. 13 D. 14 E. NOTA

7. When k is divided by 7, the result has a remainder of 3. When j is divided by 3, the result has remainder 1. What is the remainder when $k + j$ is divided by 21?

A. 4 B. 10 C. 17 D. Cannot be determined. E. NOTA

8. For how many positive integers n less than 1000 is $n! + 1$ divisible by n?

A. 0 B. 1 C. 333 D. 999 E. NOTA

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9. How many integers have 7 digits when written in binary (leading zeroes are not considered digits, so 0011000 is not a 7-digit binary number)?

A. 64 B. 128 C. 256 D. 512 E. NOTA

10. I write the integers from 2 to 400 on a piece of paper. I then circle all the primes. Then I circle all the previously uncircled numbers which are multiples of 2, 3, 5, or 7. How many numbers have no circle?

A. 12 B. 16 C. 19 D. 20 E. NOTA

11. Which of the following is the list of possible units digit of a perfect square that ends with 4 identical digits?

A. 0 only B. 0 or 4 only C. 0, 1, or 4 only D. 0, 1, 4, or 6 only E. NOTA

12. Given that p, q, and r are primes with $p < q$ and $p + q = r$, find p.

A. 2 B. 3 C. 13 D. Cannot be determined E. NOTA

13. What is the largest integer which must divide the sum of the two digit numbers AB and BA for values of the digits A and B?

A. 1 B. 3 C. 7 D. 11 E. NOTA

14. Two numbers are called *relatively prime* if their greatest common divisor is 1. How many numbers less than 120 are relatively prime to 120?

A. 32 B. 36 C. 40 D. 48 E. NOTA

15. Find the largest integer that evenly divides $n^5 - 5n^3 + 4n$ for all integers n.

A. 24 B. 60 C. 120 D. 240 E. NOTA

16. Let n be a randomly chosen two-digit positive integer. What is the probability that the fraction $6/n$ is reducible?

A. 1/2 B. 2/3 C. 4/5 D. 5/6 E. NOTA

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17. How many distinct integer solutions (x, y) are there to $x^2 - y^2 = 105$?

A. 2 B. 4 C. 8 D. 16 E. NOTA

18. What is the largest value of k such that every integer divisible by 20, 24, and 36 is divisible by k?

A. 180 B. 360 C. 4320 D. 17280 E. NOTA

19. How many perfect squares less than 500 are divisible by at least 4 distinct positive numbers?

A. 9 B. 13 C. 14 D. 22 E. NOTA

20. If m is divisible by 420 and n is divisible by 294, then what is the largest integer which divides $m + n$ for all values of m and n?

A. 6 B. 7 C. 14 D. 42 E. NOTA

21. The numbers a, b, and c are positive integers. ab is divisible by 32, bc is divisible by 42, and ac is divisible by 35. What is the largest integer which must divide abc?

A. 56 B. 112 C. 840 D. 1680 E. NOTA

22. A, B, and C are nonnegative integers less than 10. $100A + 10B + C$ is divisible by 5. $10A + C$ is divisible by 3. $10B + C$ is divisible by 4. $10A + B$ is divisible by 9. Find $A + B + C$.

A. 3 B. 9 C. 12 D. Cannot be determined E. NOTA

23. How many positive numbers less than 729 have a base 3 representation with no 2's?

A. 63 B. 81 C. 100 D. 144 E. NOTA

24. For how many positive integers k is 720 the least common multiple of 180 and k?

A. 6 B. 8 C. 12 D. 18 E. NOTA

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25. Find the maximum possible value of $x + y$ if

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{10},$$

and x and y are both integers.

A. 10 B. 121 C. 144 D. 132 E. NOTA

26. For how many positive integers k less than 100 is the statement '$2^a - 2^b$ is divisible by k for some integer values of a and b' true?

A. 0 B. 4 C. 7 D. 99 E. NOTA

27. How many solutions in positive integers (m, n) are there to the equation $3m^2 + 6n = 2003$?

A. 0 B. 3 C. 7 D. 9 E. NOTA

28. The number j^2, where j is an integer, leaves a remainder of k when divided by n $(k < n)$. How many possible pairs of values (k, n) are there with $1 < n < 10$.

A. 22 B. 23 C. 24 D. 26 E. NOTA

29. We define the set of **cycles** of an integer to be all the integers that are formed by successively moving the units digit to the front and moving all the other digits over one place to the right. For example, the set of cycles of 4275 is $\{5427, 7542, 2754, 4275\}$ and the set of cycles of 51904 is $\{45190, 04519, 90451, 19045, 51904\}$. (Note the treatment of the 0 - it doesn't disappear as we form our cycles!)

How many positive 10-digit integers n (i.e. $10^{10} > n \geq 10^9$) have the property that all members of the set of cycles of n are divisible by 11111?

A. 900009 B. 810009 C. 1110000 D. 0 E. NOTA

30. Given $f(1) = 1$, $f(2n) = f(n)$ and $f(2n + 1) = f(2n) + 1$ for all integers n, for how many values of n is $f(n) = 10$ for $n < 2003$?

A. 2 B. 5 C. 6 D. 9 E. NOTA