2004 Mu Alpha Theta National Convention
Advanced Calculus – Mu Division
For all questions, the answer "E. NOTA" means none of the above answers is correct

1. Which of the following is the derivative of $y = 4x\arcsin(3x)$?

(A) $\frac{4x}{\sqrt{1-9x^2}}$ (B) $4\arcsin(3x)$ (C) $\frac{4x}{\sqrt{1-x^2}} + 4\arcsin(3x)$

(D) $\frac{12x}{\sqrt{1-9x^2}} + 4\arcsin(3x)$ (E) NOTA

2. What is the derivative of $f(x) = (3x)^{\frac{1}{x}}$?

(A) $\frac{1-\ln(3x)}{x^2}(3x)^{\frac{1}{x}}$ (B) $-\frac{1}{x^2}(3x)^{\frac{1}{x}}$ (C) $-\frac{\ln(3x)}{x^2}(3x)^{\frac{1}{x}}$

(D) $\frac{\ln(3x) - (\ln(3x))^2}{x^3}$ (E) NOTA

3. If $y = f(4x-7)$ and $f'(x) = e^{-x^2}$, what is $\frac{dy}{dx}$?

(A) $(56-32x)e^{-16x^2+56x-49}$ (B) $(56-32x)e^{-x^2}$ (C) $-2xe^{-x^2}$

(D) $4e^{-16x^2+56x-49}$ (E) NOTA

4. Which of the following is equal to $\int 8x^2(2x+3)^{\frac{1}{3}}\,dx$?

(A) $\frac{18}{5}(2x+3)^{\frac{5}{3}} + \frac{27}{2}(2x+3)^{\frac{2}{3}} + C$

(B) $\frac{9}{2}(2x+3)^{\frac{4}{3}} + C$

(C) $\frac{3}{10}(2x+3)^{\frac{10}{3}} - \frac{18}{7}(2x+3)^{\frac{7}{3}} + \frac{27}{4}(2x+3)^{\frac{4}{3}} + C$

(D) $\frac{3}{8}(2x+3)^{\frac{8}{3}} - \frac{18}{5}(2x+3)^{\frac{5}{3}} + \frac{27}{2}(2x+3)^{\frac{2}{3}} + C$

(E) NOTA

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5. If $\frac{dy}{dx} = 2xy^2 - (yx)^2$, and $y = 1$ when $x = 0$, what is the value of *y* when $x = 1$?

(A) $\ln 2$ (B) e (C) 2 (D) 3 (E) NOTA

6. Consider a solid with a base in the *xy*-plane bounded by the *x*-axis, the *y*-axis, and the line $y = -2x + 4$. If every cross-section of this solid perpendicular to the *y*-axis is an equilateral triangle, what is the volume of this solid?

(A) $\sqrt{3}$ (B) $\frac{4\sqrt{3}}{3}$ (C) 3 (D) 4 (E) NOTA

7. A particle's velocity as a function of time is $v(t) = a + bt^2$, and at $t = 1$ its acceleration, velocity, and position are all 2. What is the total distance traveled by the particle between $t = 0$ and $t = 2$?

(A) $\frac{13}{3}$ (B) $\frac{14}{3}$ (C) $\frac{15}{3}$ (D) $\frac{16}{3}$ (E) NOTA

8. Evaluate: $\lim_{n \to \infty} \left(\sum_{k=1}^{n} \left(\frac{\pi}{4n} \cos\left(\frac{k\pi}{4n} \right) \right) \right)$

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) NOTA

9. Evaluate: $\int_{\frac{\pi}{2}}^{\pi} 3\sin^3(a)\,da$

(A) 2 (B) $\frac{\pi}{2}$ (C) $\frac{3}{2}$ (D) π (E) NOTA

10. A particle's position in the Cartesian plane is determined parametrically by
$x = 3t + 6t^2$
$y = 20 - 2t^3$.
What is its speed when $t = 1$?

(A) $9\sqrt{3}$ (B) $3\sqrt{29}$ (C) $12\sqrt{2}$ (D) 18 (E) NOTA

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11. For how many integer values of *x* does $\sum_{n=1}^{\infty} \frac{(n+1)x^9\left(\frac{1}{8}x-5\right)^n}{n^4(x-1)}$ converge?

(A) 14 (B) 15 (C) 16 (D) 17 (E) NOTA

12. A curve is defined parametrically as $x = \frac{t^2+1}{t^2}$ $y = t^3 - t^2 + 1$. Which of the following is equal to $\frac{d^2y}{dx^2}$?

(A) $\frac{15}{4}t^7 - 2t^6$ (B) $\frac{3}{4}t^6 - 15t^5$ (C) $2t^5 - 3t^6$ (D) $t^4 - \frac{3}{2}t^5$ (E) NOTA

13. What is the coefficient of the $(x-2)^2$ term of the Taylor series for $y = \frac{1}{x}$ about 2?

(A) 1 (B) $\frac{1}{2}$ (C) $-\frac{1}{4}$ (D) $\frac{1}{8}$ (E) NOTA

14. Which of the following is the Maclaurin series for $y = x^2e^{2x}$?

(A) $\sum_{n=0}^{\infty} \frac{2^n x^{n+2}}{n!}$ (B) $\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$ (C) $\sum_{n=0}^{\infty} \frac{2^{n+2} x^n}{n!}$ (D) $\sum_{n=1}^{\infty} \frac{2^n x^n}{(n-1)!}$ (E) NOTA

15. Given the polar relationship $r = 2\sin\theta$, determine $\frac{dy}{dx}$ when $\theta = \frac{\pi}{6}$.

(A) $\sqrt{3}$ (B) $\sqrt{2}$ (C) 1 (D) $\frac{\sqrt{3}}{2}$ (E) NOTA

16. What is the area of the region enclosed by the polar graph of $r = \theta$ when $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$?

(A) π (B) $\frac{4\pi}{3}$ (C) $\frac{\pi^3}{24}$ (D) $\frac{\pi}{3}$ (E) NOTA

17. If $f''(x) = 3x + 1$, $f'(1) = 2$, $f(2) = 3$ find f(x).

(A) $f(x) = 1.5x^2 + x - .5$ (B) $f(x) = 1.5x^2 + x + 2$ (C) $f(x) = .5x^3 + .5x^2 - .5x - 2$

(D) $f(x) = .5x^3 + .5x^2 - .5x + 3$ (E) NOTA

18. Which of the following is a solution to the differential equation $y' = \frac{y^2}{x^2} + \frac{y}{x}$?

(A) $y = \frac{x}{1-\ln x}$ (B) $y = \frac{x}{\ln x}$ (C) $y = \frac{\ln x}{1-x}$ (D) $y = \frac{\ln x}{x}$ (E) NOTA

19. Determine $\frac{\partial b}{\partial c}$ if $b(c,d) = 3cd^3 + c - \frac{d^2}{c}$.

(A) $4 + \frac{1}{c^2}$ (B) $3d^3 - \frac{d^2}{c^2}$ (C) $9d^2 + 1 + \frac{2d}{c^2}$ (D) $3d^3 + 1 + \frac{d^2}{c^2}$ (E) NOTA

20. Which double integral represents the area of the region in the first quadrant bounded by the graphs of $x = y^2 + 5$, $y = -2x + 13$, and the *x*-axis? Assume that $x \geq 5$.

(A) $\int_6^{29/4} \int_{\sqrt{x-5}}^{13-2x} dydx$ (B) $\int_6^{29/4} \int_{-3/2}^{1} dydx$ (C) $\int_0^1 \int_{y^2+5}^{(13-y)/2} dxdy$

(D) $\int_{-3/2}^{1} \int_6^{29/4} dxdy$ (E) NOTA

21. Evaluate: $\int_{-12}^{12} \int_{-\sqrt{144-z^2}}^{\sqrt{144-z^2}} \int_{\frac{\sqrt{z^2+y^2}}{2}}^{6} dxdydz$

(A) 432π (B) 288π (C) 256π (D) 144π (E) NOTA

22. Evaluate: $\int_0^{\frac{\pi}{3}} \left(\frac{\tan x e^{\sec x}}{\cos x} \right) dx$

(A) $e^2 - e$ (B) $e^2 - 1$ (C) e^2 (D) $\sqrt{e}$ (E) NOTA

23. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \left(\frac{2^n}{n+1} \right)$ II. $\sum_{n=1}^{\infty} \frac{3}{n}$ III. $\sum_{n=1}^{\infty} \left(\frac{\cos 2n\pi}{n^2} \right)$

(A) I only (B) II only (C) III only (D) I and II only (E) NOTA

24. Given that $w = uv - \frac{v^2}{u}$, $u = \frac{4t}{s}$, and $v = \frac{s^2}{t^3}$, find $\frac{\partial w}{\partial t}$ at (*t, s*) = (1, 2).

(A) 208 (B) 132 (C) 40 (D) 17 (E) NOTA

25. The length of the curve described by $x = 2t^2$ and $y = t^3$ for $0 \le t \le 1$ is

(A) $\frac{5}{7}$ (B) $\sqrt{5}$ (C) $\frac{\sqrt{5}}{2}$ (D) $\frac{3}{2}$ (E) NOTA

26. A particle moves on a plane curve so its position is $x = 3t^2 - 7$ and $y = \frac{4t^2 + 1}{3t}$ for t > 0. The acceleration vector of the particle at t = 2 is

(A) $\langle 6, \frac{1}{12} \rangle$ (B) $\langle 17, \frac{17}{6} \rangle$ (C) $\langle 12, \frac{47}{12} \rangle$ (D) $\langle 6, \frac{17}{6} \rangle$ (E) NOTA

27. Evaluate $\lim_{x \to 2} \left[\frac{\int_{-2}^{x} t^3 dt}{x^2 - 4} \right]$

(A) 0 (B) 2 (C) 4 (D) 8 (E) NOTA

28. Evaluate: $\int_0^1 \int_y^1 e^{x^2} dxdy$

(A) $2e - 1$ (B) e (C) $\frac{e}{2} - \frac{1}{2}$ (D) $\frac{e}{3}$ (E) NOTA

29. The water level in a cylinder is falling at a rate of 1 inch per minute. If the radius of the cylinder is 10 inches, what is the rate that the water is leaving when the volume is 500π cubic inches?

(A) $-\pi$ (B) -100π (C) -200π (D) -500π (E) NOTA

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30.

x	0	1	2	3	4	5	6
f(x)	1	2	4	1	3	2	5

Given that f is a continuous function, and if three equal subintervals are used for $0 \le x \le 6$. Which of the following is equivalent to a right-hand Riemann Sum approximation for $\int_0^6 f(x)dx$

(A) 14 (B) 17 (C) 20 (D) 24 (E) NOTA

Tiebreaker 1:

Let *R* be the region in the first quadrant bounded by the graphs of $y = x$ and $y = x^2$. Find the volume of the solid formed when *R* is revolved about the line $y = x + 5$.

Tiebreaker 2:

What is the average value of y = sin 2x over $\left[\frac{\pi}{4}, \frac{\pi}{3}\right]$

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Mu Advanced Calculus
Answers

#	Answer	#	Answer
1	D	18	A
2	A	19	D
3	D	20	C
4	C	21	B or E
5	D	22	A
6	B	23	E
7	B	24	C
8	D	25	E
9	A	26	A
10	B	27	B
11	E	28	C
12	A	29	E
13	D	30	D
14	A	TB1	$\frac{17\pi\sqrt{2}}{20}$
15	A	TB2	$\frac{3}{\pi}$
16	C	TB3	
17	C		

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1. (D) By the Product Rule, $\frac{d}{dx}4x\arcsin(3x) = 4\arcsin(3x) + 4x\left(\frac{3}{\sqrt{1-(3x)^2}}\right)$.

2. (A) Let $y = (3x)^{1/x}$. Taking natural logs of both sides and using logarithmic differentiation, we get $\ln y = \frac{\ln(3x)}{x} \to \frac{1}{y}\frac{dy}{dx} = \frac{(1/x)(x)-(1)\ln(3x)}{x^2}$, so that $\frac{dy}{dx} = (3x)^{1/x}\frac{1-\ln(3x)}{x^2}$.

3. (D) If $y = f(4x-7)$, then by the Chain Rule, $\frac{dy}{dx} = 4f'(4x-7) = 4e^{-(4x-7)^2} = 4e^{-16x^2+56x-49}$.

4. (C) Let $u = 2x+3$, so that $du = 2\,dx$, transforming the integral to $\int 8x^2(2x+3)^{1/3}\,dx = \int 8\left(\frac{u-3}{2}\right)^2 u^{1/3}\frac{du}{2} = \int (u-3)^2u^{1/3}\,du = \int u^{7/3} - 6u^{4/3} + 9u^{1/3}\,du$, which is equal to
$\frac{3}{10}u^{10/3} - \frac{18}{7}u^{7/3} + \frac{27}{4}u^{4/3} + C = \frac{3}{10}(2x+3)^{10/3} - \frac{18}{7}(2x+3)^{7/3} + \frac{27}{4}(2x+3)^{4/3} + C$.

5. (D) By separation of variables, we have $\frac{1}{y^2}dy = (2x - x^2)dx$, or after integrating both sides, $-\frac{1}{y} = x^2 - \frac{x^3}{3} + C$. Using the initial condition $(0,1)$, we find that C = –1. Thus, when *x* = 1, we get that $-\frac{1}{y} = 1^2 - \frac{1^3}{3} - 1$, or *y* = 3.

6. (B) Because the shape is an equilateral triangle, the area of each cross section is given by $A(x) = \frac{x^2\sqrt{3}}{4}$. Slicing in the *y*-direction, we see that the volume is equal to
$\int_0^4 A(x)dy = \int_0^4 \frac{x^2\sqrt{3}}{4}dy = \int_0^4\left(\frac{4-y}{2}\right)^2\frac{\sqrt{3}}{4}dy = \frac{4\sqrt{3}}{3}$.

7. (B) The acceleration is given by $a(t) = v'(t) = 2bt$. Since $v(1) = a(1) = 2$, we get the equations $a + b = 2$ and $2b = 2$; thus *b* = 1 and *a* = 1. The total distance traveled over the interval $0 \le t \le 2$ is then $\int_0^2 |v(t)|dt = \int_0^2 |1+t^2|dt = \int_0^2 1+t^2\,dt = \frac{14}{3}$.

8. (D) The limit represents a Riemann sum of $f(x) = \cos x$ on the interval $0 \le x \le \pi/4$, so the answer is $\int_0^{\pi/4}\cos x\,dx = \frac{\sqrt{2}}{2}$.

9. (A) Since the power of sine is odd, break off a sine and convert everything to cosine; this'll allow us to let $u = \cos a$. So we have
$\int 3\sin^3 a\,da = \int 3\sin^2 a \sin a\,da = -3\int (1-\cos^2 a)(-\sin a\,da) = -3\int 1-u^2\,du = u^3 - 3u$.
When $\pi/2 \le a \le \pi$, the limits change to $0 \le u \le -1$; thus, the answer is $u^3 - 3u\Big]_0^{-1} = 2$.

10. (B) The answer is $\sqrt{(x'(1))^2 + (y'(1))^2}$. Since $x'(t) = 3 + 12t$ and $y'(t) = -6t^2$, we get
$\sqrt{(x'(1))^2 + (y'(1))^2} = \sqrt{15^2 + (-6)^2} = 3\sqrt{29}$.

11. (B) The factor $\dfrac{x^9}{x-1}$ will not affect convergence or divergence of the series; it suffices to test
$\sum_{n=1}^{\infty} \dfrac{(n+1)}{n^4}\left(\dfrac{x}{8} - 5\right)^n$ for convergence. By the Ratio Test, the series converges when
$\lim_{n\to\infty}\left(\dfrac{(n+2)}{(n+1)^4}\dfrac{n^4}{(n+1)}\left|\dfrac{x}{8} - 5\right|\right) = \left|\dfrac{x}{8} - 5\right| < 1$, or $32 < x < 48$. There are 15 integers on this interval.

12. (A) We have $\dfrac{dy}{dx} = \dfrac{3t^2 - 2t}{-2/t^3} = -\dfrac{3t^5}{2} + t^4$, and so
$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt} = \frac{-\frac{15t^4}{2} + 4t^3}{-2/t^3} = \frac{15}{4}t^7 - 2t^6.$$

13. (D) The coefficient is equal to $\dfrac{y''(2)}{2!} = \dfrac{2/2^3}{2} = \dfrac{1}{8}$.

14. (A) Since $e^x = \sum_{n=0}^{\infty}\dfrac{x^n}{n!}$, we have $e^{2x} = \sum_{n=0}^{\infty}\dfrac{(2x)^n}{n!} = \sum_{n=0}^{\infty}\dfrac{2^n x^n}{n!}$, so
$x^2 e^{2x} = x^2\sum_{n=0}^{\infty}\dfrac{2^n x^n}{n!} = \sum_{n=0}^{\infty}\dfrac{2^n x^{n+2}}{n!}$.

15. (A) When $\theta = \pi/6$, the corresponding value of *r* is $2\sin(\pi/6) = 1$. Using the conversion factors $x = r\cos\theta$ and $y = r\sin\theta$, we find the Cartesian coordinates of the point of interest to be $x = 1\cos(\pi/6) = \sqrt{3}/2$ and $y = 1\sin(\pi/6) = 1/2$. Given the equation $r = 2\sin\theta$, convert to Cartesian mode by going $r^2 = 2r\sin\theta \to x^2 + y^2 = 2y$. Differentiating this implicitly, we get $y' = x/(1-y)$; the answer is $(\sqrt{3}/2)/(1-1/2) = \sqrt{3}$.

16. (C) By the standard formula, the area is $\dfrac{1}{2}\int_{-\pi/2}^{\pi/2}\theta^2\,d\theta = \dfrac{\pi^3}{24}$.

17. (C) $f''(x) = \int f'''(x) = \int (3x+1)dx = 1.5x^2 + x + C$ $f''(1) = 1.5 + 1 + C = 2$ $C = -.5$
$f''(x) = 1.5x^2 + x - .5$ $f'(x) = \int f''(x) = \int (1.5x^2 + x - .5)dx = .5x^3 + .5x^2 - .5x + C$

f(2) → 4 + 2 – 1 + C = 3 C = -2 $f(x) = .5x^3 + .5x^2 - .5x - 2$

18. (A) Let $v = \frac{y}{x}$; then $y = vx$ and $\frac{dy}{dx} = x\frac{dv}{dx} + v$. Plug all of these into the differential equation to get $x\frac{dv}{dx} + v = v^2 + v$, or $\frac{1}{v^2}dv = \frac{1}{x}dx$, or $\frac{-1}{v} = -\frac{x}{y} = \ln x + C$. Solving for *y*, we get $y = \frac{x}{-C - \ln x}$. The answer given in choice A is when *C* = –1.

19. (D) Treating *d* as a constant, we find that $\frac{\partial b}{\partial c} = 3d^3 + 1 + \frac{d^2}{c^2}$.

20. (C) By inspection, the answer is choice C.

21. (B) Drawing the region of integration, we see that it's a cone with a circular base of radius 12 and height 6. The integral in question is the volume of this region, which is $\frac{\pi}{3}(12)^2(6) = 288\pi$.

22. (A) u = secx du = secx tanx dx when x = 0 u = sec 0 = 1; when $x = \frac{\pi}{3}$

$$u = \sec\frac{\pi}{3} = 2 \qquad \int_0^{\frac{\pi}{3}}\left(\frac{\tan x e^{\sec x}}{\cos x}\right)dx = \int_0^{\frac{\pi}{3}}(\sec x \tan x e^{\sec x}) = \int_1^2 e^u du = e^u\Big|_1^2 \; e^2 - e$$

23. (D) I. Divergent II. Divergent III. Convergent

24. (C) By the Chain Rule, $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial u}\frac{\partial u}{\partial t} + \frac{\partial w}{\partial v}\frac{\partial v}{\partial t} = \left(v + \frac{v^2}{u^2}\right)\left(\frac{4}{s}\right) + \left(u - \frac{2v}{u}\right)\left(\frac{-3s^2}{t^4}\right)$. When $(t, s) = (1, 2)$, then $(u, v) = (2, 4)$. Plug all these numbers in to get 40.

25. (B) The length =

$$\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^1 \sqrt{36t^4 + 9t^4}\, dt = \int_0^1 \sqrt{45t^4}\, dt = 3\sqrt{5}\left[\frac{t^3}{3}\right]_0^1 = \sqrt{5}$$

26. (C) Find the second derivative $x' = 6t$ $x'' = 6$ $y = \frac{4}{3}t + \frac{1}{3}t^{-1}$ $y' = \frac{4}{3} - \frac{1}{3}t^{-2}$ $y'' = \frac{2}{3}t^{-3}$

$$x''(2) = 6 \qquad y''(2) = \frac{2}{3}\left(\frac{1}{8}\right) = \frac{1}{12} \qquad \langle 6, \frac{1}{12}\rangle$$

27. (B) L'Hopital's Rule (form $\frac{0}{0}$) $\lim_{x\to 2}\frac{x^3}{2x} = 2$ (use 2nd fundamental theorem of calculus to find derivative of the numerator)

28. (C) The integral is impossible to evaluate at the current order of integration. If we switch the order to *dy dx*, the bounds change to $0 \le y \le x$ and $0 \le x \le 1$, making

$$\int_0^1\int_y^1 e^{x^2}\,dx\,dy = \int_0^1\int_0^x e^{x^2}\,dy\,dx = \int_0^1 xe^{x^2}\,dx = \frac{e-1}{2}.$$

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29. (B) $\frac{dh}{dt} = -1$ V=$\pi r^2 h$ r=10 (is constant) so V=$100\pi h$ $\frac{dv}{dt} = 100\pi \frac{dh}{dt} = -100\pi$
(note you don't use 500π)

30. (D) Plot the points, draw the rectangles, find the areas and add 8 + 6 + 10 = 24.

Tiebreaker Solutions:

1. Let *A* be the area of the region; thus $A = \int_0^1 x - x^2\,dx = \frac{1}{6}$. We proceed to calculate the center of mass $(\bar{x}, \bar{y})$ of *R*. We have $\bar{x} = \frac{m_y}{A} = \frac{\int_0^1 x(x-x^2)\,dx}{1/6} = \frac{1}{2}$ and $\bar{y} = \frac{m_x}{A} = \frac{\frac{1}{2}\int_0^1 (x)^2 - (x^2)^2\,dx}{1/6} = \frac{2}{5}$. The distance *r* from the centroid to the line $x - y + 5 = 0$ is given by $\frac{\left|\frac{1}{2}(1) + \frac{2}{5}(-1) + 5\right|}{\sqrt{1^2 + (-1)^2}} = \frac{51\sqrt{2}}{20}$. By the Theorem of Pappus, the volume is $2\pi r A = 2\pi\left(\frac{51\sqrt{2}}{20}\right)\left(\frac{1}{6}\right) = \frac{17\pi\sqrt{2}}{20}$.

2. Answer: $f(c) = \frac{3}{\pi}$

   Solution:

   $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$ $\quad$ f(c)=$\frac{1}{\frac{\pi}{3} - \frac{\pi}{4}}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin 2x\,dx$ $\quad$ u = 2x du = 2dx $\frac{1}{2}du = dx$

   when x=$\frac{\pi}{4}$ u=$\frac{\pi}{2}$ $\quad$ when x=$\frac{\pi}{3}$ u=$\frac{2\pi}{3}$ $\quad$ f(c)=$\frac{1}{\frac{\pi}{12}} \cdot \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \sin u\,du$ $\quad$ f(c)=$\frac{6}{\pi}\left[-\cos u\right]_{\frac{\pi}{2}}^{\frac{2\pi}{3}}$

   f(c)=$\frac{6}{\pi}\left[-\cos\frac{2\pi}{3} + \cos\frac{\pi}{2}\right]$ $\quad$ f(c)=$\frac{6}{\pi}\left(\frac{1}{2}\right) = \frac{3}{\pi}$