1. A Let \( r \) be the radius of the smaller circle, \( R \) be the radius of the larger circle. \( \frac{1}{2} \) of the tangent chord forms a right triangle with sides \( r \), \( R \), and 4. Our shaded area is equal to: 
\[
\frac{1}{2} \pi r^2 - \frac{1}{2} \pi R^2 = \frac{1}{2} \pi (R^2 - r^2) = \frac{1}{2} \pi \left( \left(\frac{7}{3}\right)^2 - \left(\frac{5}{3}\right)^2 \right) = \frac{16}{3} \pi
\]

2. C 1/3 of the distance in the \( x \) direction is \( \frac{2}{3} \) of \( \frac{2}{3} \) units, and 1/3 of the distance along the \( y \) direction is \( \frac{3}{4} \). Therefore, the point in question is:
\[
\left( \frac{2}{3} \cdot \frac{2}{3}, \frac{3}{4} \right) = \left( \frac{4}{9}, \frac{3}{4} \right)
\]

3. B \( r_{\text{circum}} = \frac{a \cdot b \cdot c}{4 \cdot \text{Area}} = 6.5 \), \( r_{\text{in}} = \frac{\text{Area}}{\text{perimeter}} = \frac{30}{15} = 2 \), \( \frac{r_{\text{circum}}}{r_{\text{in}}} = 3.25 = \frac{13}{4} \)

4. C This equation can be factored as \( (2x - y + 1)^2 = 0 \) which is a single line

5. A Choose a point on the second line, say \((x, y)\). The shortest distance from a point \((x_0, y_0)\) to a line \(Ax + By + C = 0\) is found by:
\[
\frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} = \frac{39}{13} = 3
\]

6. A The numerator can be rewritten \( 4x - 3 = 4(x - 1) + 4 - 3 = 4(x - 1) + 1 \). Therefore our hyperbola can be found by:
\[
y = \frac{4(x - 1) + 1}{x - 1} = 4 + \frac{1}{x - 1} \quad \text{and} \quad y - 4 = \frac{1}{x - 1}, \quad \text{which has center} \,(1,4)
\]

7. B \( g(x) \) will look as if \( f \) was rotated about the line \( y = x \), and \( h(x) \) resembles this \( g(x) \) rotated about the \( x \)-axis, which leaves \( h(x) = -g(x) \).

8. E The value of the cosine of the angle in between the vectors \( \mathbf{a} \) and \( \mathbf{b} \) is equal to:
\[
\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} = \frac{7}{25} < 1 \quad \text{and} \quad \det \begin{vmatrix} -1 & -i & -1 \\ i & 0 & i \\ 1 & i & 1 \end{vmatrix} = 0.
\]

9. D \( x = \frac{-b}{2a} = -4 \quad y = (-4)^2 + (8)(-4) - 7 = -23 \)

10. A Each of the fifth roots of \(-2 \) lie on a circle around the origin with radius \( \sqrt{2} \), where the absolute value of each point on the circle is \( \sqrt{2} \). Therefore, the sum of 5 fifth roots is \( 5 \sqrt{2} \).

11. B Substituting \( r^2 = x^2 + y^2 \) after squaring both sides, and using \( \sec(2\theta) = \frac{1}{\cos(2\theta)} = \frac{1}{\cos^2 \theta - \sin^2 \theta} \)
\[
\cos \theta = \frac{x}{r}, \quad \sin \theta = \frac{y}{r}, \quad \text{we get} \quad x^2 - y^2 = 1, \quad \text{where} \quad a = 1, b = 1, c = \sqrt{2}.
\]
17. Dividing our octagon into 8 triangles, each has base \( s \) and height \( \frac{135\tan^{-2}s}{2} \). \( \tan\left(\frac{135}{2}\right) = \frac{\sqrt{2}}{2} \), so 8 of the triangles has area \( 2s^2(\sqrt{2}+1) \).

18. \( 4\sin\theta\cos\theta = 2\sin(2\theta) \) For \( r = a\sin(b\theta) \) where \( b \) is even, the number of petals, \( p = 2b \).

19. The volume of the solid described by 3 vectors \( (a, b, c) \) can be found by \( a \cdot (b \times c) \). Therefore, \( b \times c = (2,0,0) \) and \( a \cdot (b \times c) = (1,1,0) \times (2,0,0) = 2 \).

20. The two points described are the foci of the conic section. The fact that the positive difference remains constant implies this is a hyperbola. The center is 2/3 way between the foci \( (0,1) \), making \( a=2 \). \( c=3 \) is the distance between the center and a focus, and \( c^2 = a^2 + b^2 \), so \( b^2 = 5 \).

21. The distance between the latera recta (the width of the rectangle in question) is \( 2c \), where \( c = \sqrt{a^2 - b^2} = 2 \).

The height of the rectangle is the length of a latus rectum, \( h = \frac{2b^2}{a} = \frac{2\cdot9}{\sqrt{13}} \). The area of the rectangle is the product, \( h \times w \).

22. The eccentricity of all parabolas is 1.

23. The slopes of the asymptotes are found as \( \pm \frac{b}{a} = \pm \frac{\sqrt{45}}{\sqrt{5}} = \pm 3 \).

24. The domain of \( \log_a x \) does not allow for \( x \) to be negative; the function is asymptotic for small values of \( x \) approaching \( x = 0 \). As \( x \) increases, \( x - \log_a 256 \) gets infinitely large, yielding \( y = 0 \). Rearranging the denominator, \( \log x^4 - \log 256 \) yields \( x=4 \) an asymptote.

25. Using Green’s Theorem for polygons:
\[
(0 \cdot 4) + (1017 \cdot 568) + (79 \cdot 0) - (0 \cdot 1017) - (4 \cdot 79) - (568 \cdot 0)
\]
all divided by 2 = 288670.

26. This is a line that passes through the origin, therefore can be represented by \( \theta = a \), where \( a \) is the arctangent of the slope.

27. Start with the equation \( (x-3)^2 + (y+1)^2 = 5^2 \) and expand & rearrange to get C.

28. For a hyperbola, \( e = \frac{c}{a} = \frac{a^2 + b^2}{a} \), and therefore \( \frac{1 + \frac{\sqrt{5} - 1}{2}}{1} = \frac{1 + \sqrt{5}}{2} \).

29. We can rearrange \( \log y = t \log \sqrt{2} \) to become, \( t = \log_{\sqrt{2}} y^2 \), so \( x = 2^t = y^2 \). But \( y > 0 \), and \( x > 0 \), so \( \sqrt{x} = y \).

30. The five platonic solids are: tetrahedron, cube, octahedron, dodecahedron, and icosahedron with sides of 4, 6, 8, 12, and 20, respectively.