1. B
2. A
3. C
4. D
5. C
6. C
7. A
8. B
9. A
10. D
11. B
12. A
13. B
14. B
15. D
16. C
17. A
18. A
19. C
20. D
21. Thrown Out
22. A
23. C
24. A
25. B
26. D
27. C
28. A
29. D
30. B

1. Solving the equations $4c+7a=165$ and $a+c=27$ gives c=8. **B**

2. The area of a triangle spanned by the vectors <*a*,*b*> and <*b*,*c*> is half of $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Which equals $19/2$. **A**

3.

4. Jack can complete $1/14$ of the test in a day and together they can complete $1/8$ of the test in a day. So Josh alone can complete $1/8 - 1/14 = 3/56$ of the test in a day: it will take him $\frac{56}{3}$ days. **D**

5. Given equal heights and volumes in a 3:1 ratio, the radii are in a $\sqrt{3}:1$ ratio. So we have two equations and two unknowns: $r_1 = r_2\sqrt{3}$, $r_1 = r_2 + 2$, giving $r_1 = 3+\sqrt{3}$. The area of the base is then: $(12+6\sqrt{3})\pi$ **C**

6. In *cis* notation we have: $\left(\sqrt{2}cis45\right)^{2005} + \left(\sqrt{2}cis-45\right)^{2005}$. Adding, the angles of the vectors will cancel as they are opposite and have the same factor. So for the resultant sum we have: $2*\left(-\frac{\sqrt{2}^{2005}}{\sqrt{2}}\right) = -2^{1003}$ **C**

7. The ship's path is an equilateral triangle. The bearing is then $60°$ past the original $45°$ making it $105°$. **A**

8. Using *cis* notation again we have: $\left(2cis30\right)^m = \left(\sqrt{2}cis-45\right)^n$. So given that $n=2m$, it is just a matter of lining up the angles. This first occurs with $m=3, n=6$. **B**

9. Each consonant occurs once, so there are $4c2=6$ ways of picking them. For vowel pairs there are two choices: 'A,O' and 'O,O'. For each consonant pair there are $4!=24$ ways of arranging the letters given the 'A,O' vowel pair and $\frac{4!}{2}=12$ ways given the 'O,O' pair making $6*(12+24)=216$ ways total. **A**

10. Calling the distance from the foot of the ladder on the ground to the base of the shorter fence *x* and the height of the point on the wall where the ladder will reach *h* we can write a proportion: $\frac{6}{x}=\frac{h}{x+2}$ which gives: $h = \frac{6x+12}{x}$. The length of the ladder, *l*, can be expressed as: $l^2 = h^2 + \left(x+2\right)^2 = \left(\frac{6(x+2)}{x}\right)^2 + \left(x+2\right)^2 = \left[\left(\frac{6}{x}\right)^2+1\right]\left(x+2\right)^2$. Evaluating values for x near 4 quickly shows that the minimum length is 11. **D**

11. Let $x=$ the amount invested at 9%
$2x=$ the mount invested at 6%
$25,000-3x$ = the amount invested at 8%

$$.09x+2\left(.06x\right)+.08\left(25,000-3x\right)=1850$$
$$.09x+.12x+2,000-.24x=1850$$
$$-.03x=-150$$
$$x=5000$$

Answer choice B.

12. This problem can be seen as 20 markers being divided by four placers and hence into 5 groups. Each section corresponds to the number of problems done by that person, giving $19c4$, or 3876 ways. **A**

13. Going piece by piece we have: $A=3, B=\pi, C=\frac{1}{2}, D=4$. This gives $\dfrac{3+\pi+\frac{1}{2}}{4+1}=\dfrac{2\pi+7}{10}$. **B**

14. The sum the roots of $x^3-27=0$ is clearly 0, and the only real root is 3: $0-3=-3$. **B**

15. The initial term in the infinite summation is the probability that Aria misses and James hits. The ratio is then the probability that all three miss in succession. Hence the probability that James wins is:
$$\frac{2/3 * 1/3}{1-\left(2/3\right)^3}=\frac{2/9}{1-8/27}=\frac{2/9}{19/27}=6/19$$. **D**

16. Finding the eigen values is equivalent to solving $\det|A-\lambda I|=0$, or $\begin{vmatrix}1-\lambda & 4 & 7\\ 0 & -3-\lambda & 6\\ 0 & 0 & 7-\lambda\end{vmatrix}=0$. This can be solved by expanding the determinant and solving for the largest root of the cubic equation, or seeing that for the determinant to be non-zero all three diagonal elements must be non-zero. Hence, they are the eigen values and 7 is the largest. **C**

17. If the coating has triple the volume, then the new sphere has a volume quadruple of the original. The new radius is then $2^{2/3}$ times as big as the original. Solving for the original radius from the volume gives it to be 6. So the new radius is $6*2^{2/3}$, giving the new surface area to be $288\pi 2^{1/3}$. **A**

18. The maximum of $2(\sin x+\cos x)$ occurs at $x=\pi/4$. The ratio is then: $\dfrac{2\sqrt{2}}{1/2}=4\sqrt{2}$. **A**

19. This is simply a matter of summing the vectors: $4(\cos 30°, \sin 30°)$ and $6(\cos 135°, \sin 135°)$; or $(2\sqrt{3},2)+(-3\sqrt{2},3\sqrt{2})=(2\sqrt{3}-3\sqrt{2},2+3\sqrt{2})$. The magnitude is then: $\left(12-12\sqrt{6}+18\right)+\left(4+12\sqrt{2}+18\right)=52+12\sqrt{2}-12\sqrt{6}$ **C**

20. The values can be plugged into the equation, or it can factored after seeing that $t=-1$ is a root to: $(t-8)(t+8)(t+1)$. The only positive root is $t=8$. **D**

21. This is essentially a harmonic mean of the four different interval times. So solving for the reciprocal of the average of the reciprocals we have: $\dfrac{4*5*6*10*8}{5*6*10+5*6*8+6*8*10+5*10*8}=\dfrac{480}{71}$. **B**

22. Descartes' Rule of Signs here involves counting the number of sign changes in $P(-x)=x^6+5x^5+3x^4-6x^3-8x^2+20x+24$. Two sign changes implies at most 2 negative roots.**A**

23. Letting *a*,*b*,*c* be the three numbers we have $ab=27, bc=32, ac=24$, multiplying the three together we have $(abc)^2=27*32*24$, so $abc=\sqrt{27*32*24}=144$. **C**

24. The 8 inch diameter pizza is an annulus of inner radius 2 and outer radius 4 giving it an area of 12π which implies that 6π square units of pizza can feed one student. The larger pizza is an annulus of inner radius 3 and outer radius 6, giving it an area of 27π. As there are two of these pizzas, and each student needs 6π square units, $\dfrac{54\pi}{6\pi}=9$ students can be fed. **A**

25. Given that this question is being answered correctly, the problem can be reduced to trying to score 22 points out of 29 questions. This is best then solved by enumerating the possibilities, of which there are 5.**B**

26. The remainder can be found simply by plugging in the value of *x* which makes the divisor equal to zero.
$(-2)^{2005}-3(-2)^{1002}+4(-2)^{1000}+(-2)^2+(-2)-1=-2^{2005}-3*2^{1002}+2^{1002}+4-2-1$

$= -2^{2005} - 2^{1003} + 1$. **D**

27. $A = \sqrt{6 + A}, A = 3$; $B = \sqrt{9 - B}, B = \dfrac{\sqrt{37} - 1}{2}$. So $AB = \dfrac{3\left(\sqrt{37} - 1\right)}{2}$. **C**

28. The parabola can be written as: $y = \dfrac{2}{45}\left(900 - x^2\right)$. From there the point $x = 15$ is plugged in as the truck is 30 feet wide and it is found that the truck can be 30 feet tall. **A**

29. The ball initially falls 48 feet before hitting that ground. After that point its travel can be seen as two infinite geometric sequences (one for the ball moving up, the other for it falling). These sequences have initial terms of 36 and ratio ¾, giving them a total final sum of 288. Adding the first 48 feet gives 336. **D**

30. Expressing the roots in the form *a*cis *b*, this question asks for how many of the roots are $-\dfrac{\pi}{4} < b < \dfrac{\pi}{4}$ or $\dfrac{3\pi}{4} < b < \dfrac{5\pi}{4}$. The roots are of the form $2^{1/15} cis\left(k\dfrac{2\pi}{15}\right)$ with $k \in [0,14]$. Counting the valid roots finds that there are 7. **B**