1) A
2) B
3) D
4) C
5) D
6) C
7) C
8) A
9) AorB
10) C
11) B
12) E
13) C
14) A
15) B
16) C
17) C
18) D
19) A
20) C
21) D
22) D
23) B
24) C
25) A
26) A
27) E
28) D
29) A
30) D

1) $(3+i)+(9-3i)-2(7+4i) = (3+9-14)+(i-3i-8i) = -2-10i$ **A**

2) $(3+i)(2-3i) = 6-9i+2i+3 = 9-7i$ **B**

3) $\dfrac{3+i}{2-3i} = \dfrac{(3+i)(2+3i)}{2^2+3^2} = \dfrac{3+11i}{13} = \dfrac{3}{13}+\dfrac{11}{13}i$ **D**

4) $6-6i\sqrt{3} = 12\text{cis}(300^\circ)$ $\quad (6-6i\sqrt{3})^5 = 12^5\text{cis}(5(300^\circ)) = 12^5\text{cis}(1500^\circ)$ $\quad \dfrac{1500}{360} = 4\dfrac{60}{360}$

$(6-6i\sqrt{3})^5 = 12^5\text{cis}(60^\circ) = 12^5\cos(60^\circ)+12^5 i\sin(60^\circ) = 12^5\left(\dfrac{1}{2}\right)+12^5\left(\dfrac{\sqrt{3}}{2}\right)i$ $\quad \dfrac{a}{b} = \dfrac{1/2}{\sqrt{3}/2} = \dfrac{\sqrt{3}}{3}$ **C**

5) $\text{Re}\{(\sqrt{2}+2i)(-i\sqrt{2}+1)(2\sqrt{2}-i)\} = \text{Re}\{(3\sqrt{2})(2\sqrt{2}-i)\} = \text{Re}\{12-3i\sqrt{2}\} = 12$ **D**

6) $\text{Im}\{(1+i)^2 \cdot e^{-i\pi/4}\} = \text{Im}\left\{(2i)\left(\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{2}}{2}i\right)\right\} = \text{Im}\{\sqrt{2}+i\sqrt{2}\} = \sqrt{2}$ **C**

7) $z_1 = a+bi$, $z_2 = c+di$ $\quad z_1+z_2 = (a+c)+(b+d)i \Rightarrow a+c=0$ $\quad$ (1) $a=-c$

$z_1^{\,2}+z_2^{\,2} = (a^2-b^2+c^2-d^2)+(2ab+2cd)i \Rightarrow 2ab+2cd=0$ $\quad$ (2) $b=d$ if $a\neq 0$ and $c\neq 0$

I) $\overline{z_2} = c-di$, but from (2) $b=d$ — FALSE

II) $(a+bi)-(c+di) = (a-c)+(b-d)i$ $\quad$ From (2) $b-d=0$ — TRUE

III) $a=c$ only when $a=c=0$, which cannot happen. — TRUE

IV) $|z_1| = \sqrt{a^2+b^2}$ $\quad |z_2| = \sqrt{c^2+d^2}$ From (1) $a^2=c^2$ and from (2) $b^2=d^2$ — TRUE

II., III., and IV. are true. **C**

8) $z = \dfrac{-2\pm\sqrt{4-4(16)}}{2} = -1\pm i\sqrt{15}$ $\quad (-1+i\sqrt{15})^2+(-1-i\sqrt{15})^2 = (-14-2i\sqrt{15})+(-14+2i\sqrt{15}) = -28$ **A**

9) $z_2 - z_1 = -4+4i$ $\quad \theta = \tan^{-1}(4/-4) = 135^\circ$ **B**

10) $1+(1-i)+(1-i)^2+(1-i)^3 = \dfrac{1-(1-i)^4}{1-(1-i)}$ $\quad (1-i)^4 = (2i)^2 = -4$ $\quad \dfrac{1-(-4)}{i} = -5i$ $\quad |0-(-5)| = 5$ **C**

11) The six sixth roots of unity are $\text{cis}(0^\circ)$, $\text{cis}(60^\circ)$, $\text{cis}(120^\circ)$, $\text{cis}(180^\circ)$, $\text{cis}(240^\circ)$, and $\text{cis}(300^\circ)$. These correspond to 1, $\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$, $-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$, -1, $-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$, $\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$. B is not on the list **B**

12) A complex number is a number of the form $a+bi$ where a and b are real numbers. All four can be represented in this form. **E**

13) $z = 32\text{cis}(45^\circ)$ $\quad$ The five fifth roots of z are $2\text{cis}\left(\dfrac{45+360\cdot n}{5}\right)$ $2\text{cis}(9^\circ)$, $2\text{cis}(81^\circ)$, $2\text{cis}(153^\circ)$, $2\text{cis}(225^\circ)$, and $2\text{cis}(297^\circ)$. C is not on the list **C**

14) $f(2+i) = i(2+i)^3+2(2+i)^2-3\overline{(2+i)}+2i = i(2+11i)+2(3+4i)-3(2-i)+2i =$
$(-11+6-6)+(2i+8i+3i+2i) = -11+15i$ **A**

15) I) $|z_2| = \sqrt{(a^2-b^2)^2+(2ab)^2} = \sqrt{a^4+2a^2b^2+b^4} = a^2+b^2 = |z_1|^2$ — TRUE

II) Let $z_1 = r\text{cis}(\theta) \Rightarrow z_2 = r^2\text{cis}(2\theta)$. The angle of z_2 is twice the angle of z_1 — TRUE

III) $z_2 = a^2-b^2+2iab$. If $b>a$, then $a^2-b^2<0$ — FALSE $\quad$ IV) $\dfrac{1}{z_2} = \dfrac{\overline{z_2}}{|z_2|^2} = \dfrac{\overline{z_1^{\,2}}}{|z_1|^4}$ — FALSE

I., and II. are true. **B**

16) $\frac{1}{i}+\frac{2}{i^2}+\frac{3}{i^3}+\frac{4}{i^4}=-i-2+3i+4=2+2i$ **C**

17) $\|-6-8i\|=\sqrt{(-6)^2+(-8)^2}=\sqrt{100}=10$ **C**

18) $i^6=i^4\cdot i^2=-1;\ i^{29}=i^{28}\cdot i=i;\ i^{2006}=i^{2004}\cdot i^2=-1$ $\quad -1+i-1=-2+i$ **D**

19) $N=0;\ \Pi=1$ $\quad N=1;\ \Pi=-i$ $\quad N=2;\ \Pi=-1$ $\quad N=3;\ \Pi=i$ $\quad N=4;\ \Pi=1$

The pattern repeats indefinitely. $\Pi=i^{-N}$ **A**

20) $(1-i-1+i)+(1-i-1+i)+\ldots+(1-i-1+i)+1-i-1=0+0+\ldots+0+1-i-1=-i$ **C**

21) $(5-6i)(2i+1)=(5+12)+(10i-6i)=17+4i$ $\quad \operatorname{Re}\{17+4i\}=17$ **D**

22) $\begin{vmatrix} x & -1 & 2 \\ 0 & i & 2 \\ -x & i & 2i \end{vmatrix}=x\begin{vmatrix} i & 2 \\ i & 2i \end{vmatrix}-x\begin{vmatrix} -1 & 2 \\ i & 2 \end{vmatrix}=x(-2-2i)-x(-2-2i)=0$ The determinant =0 for all values of x. **D**

23) There are $\binom{4}{2}=6$ different pairs of squares. $\{-2,\ 300\}$ is the only pair with a real product Prob = $\frac{1}{6}$ **B**

24) Both roots to the polynomial are complex. The sum is $-\frac{b}{a}=-\frac{-2}{1}=2$ **C**

25) $-iz=y-ix$ which is a rotation by 90° clockwise. **A**

26) $\sum_{n=0}^{5} n\cdot i^n=0+i-2-3i+4+5i=(-2+4)+(1-3+5)i=2+3i$ **A**

27) Let $z=a+bi$, where a and b are real numbers

I.) $f(-z)=\|-a-bi\|=\sqrt{(-a)^2+(-b)^2}=\sqrt{a^2+b^2}=\|a+bi\|=f(z)$ TRUE

II.) $f(\bar{z})=\|a-bi\|=\sqrt{(a)^2+(-b)^2}=\sqrt{a^2+b^2}=\|a+bi\|=f(z)$ TRUE

III.) $f(z)=\sqrt{a^2+b^2}$, which is always real TRUE

IV.) $f(z)=0=\sqrt{a^2+b^2}$ $\quad a^2+b^2=0\Rightarrow a=0$ and $b=0$ TRUE

V.) $f(z^2)=\|a^2-b^2+2iab\|=\sqrt{(a^2-b^2)+(2ab)^2}=\sqrt{a^4+2a^2b^2+b^4}=a^2+b^2=\|a+bi\|^2$ TRUE

5 statements are true. **E**

28) Two more roots must be $z_3=5+6i$ and $z_4=2-3i$. The fifth root must be real.

I.) TRUE II.) FALSE III.) TRUE IV.) TRUE **D**

29) $\sqrt{-2}\times\sqrt{-8}\times\sqrt{-16}=(i\sqrt{2})(2i\sqrt{2})(4i)=(4)(2)(2)(i^3)=-16i$ **A**

30) Using an Augmented Matrix: $\begin{bmatrix} 1 & -i & 2i & -1 \\ i & 2 & 2-i & 3i \\ 1 & 0 & 1 & 1 \end{bmatrix}\rightarrow\begin{bmatrix} 1 & -i & 2i & -1 \\ 0 & 1 & 4-i & 4i \\ 0 & i & 1-2i & 2 \end{bmatrix}\rightarrow\begin{bmatrix} 1 & -i & 2i & -1 \\ 0 & 1 & 4-i & 4i \\ 0 & 0 & -6i & 6 \end{bmatrix}$

$z=\frac{6}{-6i}=i$ $\quad y+(4-i)z=4i\Rightarrow y=4i-(4i+1)=-1$ $\quad x-iy+2iz=-1\Rightarrow x=i(-1)-2i(i)-1=1-i$ **D**