1)	B
2)	D
3)	A
4)	B
5)	D
6)	C
7)	A
8)	D
9)	A
10)	E
11)	C
12)	A
13)	Thrown Out
14)	B
15)	D
16)	C
17)	B
18)	C
19)	C
20)	A
21)	B
22)	C
23)	C
24)	A
25)	E
26)	C
27)	D
28)	B
29)	C
30)	D

1) Prob(2 balls different color)= 1–Prob(2 balls same color)= $1-\left((4/10)(3/9)+(3/10)(2/9)+(2/10)(1/9)\right)=7/9$ **B**

2) A parabola, ellipse, & circle all have eccentricities ≤ 1. I is a hyperbola, II is a circle, III is an ellipse, IV is a parabola. II, III, & IV have eccentricities less than or equal to 1. **D**

3) $x=\sum_{n=0}^{\infty}\frac{2^{n+1}}{3^n}=2\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n=2\frac{1}{1-2/3}=6$, $\quad y=\sum_{n=1}^{\infty}\frac{5^{n-1}}{6^{n-1}}=\frac{6}{5}\sum_{n=1}^{\infty}\left(\frac{5}{6}\right)^n=\frac{6}{5}\frac{5/6}{1-5/6}=6$ **A**

4) $723_8=7(64)+2(8)+3=467_{10}, 124_5=25+2(5)+4=39_{10}, 723_8+124_5=506_{10}, 506_{10}=622_9$ **B**

5) $g(x)=2\left[1-2\sin^2(x)\right]-2\left[3\sin(x)-4\sin^3(x)\right]+1=8\sin^3 x-4\sin^2 x-6\sin x+3=0$ Let $y=\sin x$
$8y^3-4y^2-6y+3=(2y-1)(4y^2-3)=0\Rightarrow y=1/2,\pm\sqrt{3}/2$ $\sin x=1/2\Rightarrow x=\pi/6,5\pi/6$;
$\sin x=\sqrt{3}/2\Rightarrow x=\pi/3,2\pi/3$; $-\sqrt{3}/2$ does not contribute any roots $\pi/6+\pi/3+2\pi/3+5\pi/6=2\pi$ **D**

6) $a_n=2^n-1$, $n=1,2,3...$ $\quad \sum_{n=1}^{20}(2^n-1)=\sum_{n=1}^{20}2^n-\sum_{n=1}^{20}1=\frac{2^1-2^{21}}{1-2}-20=2^{21}-22$ **C**

7) $(2/x+1-x^2)^5=(2/x+1)^5+(5)(2/x+1)^4(-x^2)+...$ (the terms after these will not have any constant terms in them). From the first term, the constant term is 1. Constant term for the second part is $-5x^2(6)(2/x)^2=-120$
$1-120=-119$ **A**

8) $f(x)=\frac{(x+1)^2(x-2)}{(x+3)(x-1)}$ Vertical asymptotes at I) $x=-3$ and IV) $x=1$. **D**

9) $z=-16\sqrt{2}-16i\sqrt{2}=32\text{cis}(225^\circ)$ $z^{1/5}=2\text{cis}\left(\left[225^\circ+360^\circ n\right]/5\right)=2\text{cis}(45^\circ)$ or $2\text{cis}(117^\circ)$ or $2\text{cis}(189^\circ)$ or $2\text{cis}(261^\circ)$ or $2\text{cis}(333^\circ)$; $2\text{cis}(45^\circ)$ matches **A**

10) $1-e^x\ge 0\Rightarrow x\le 0$ f is continuous and monotone decreasing. $f(0)=0$, $\lim_{x\to\infty}f(x)=0$ f takes on the value at x = 0, but not at infinity Range = $[0,1)$ **E**

11) A cardioid has the form $r=a(1+\cos\theta)$ where a is a constant. **C**

12) There are 7 possibilities: a) a four in the last position and any number other than four in the first two ($=8\cdot 9$), b) a four in the second position, any number other than four in the first position, and an even number other than four in the last position ($=8\cdot 4$), c) a four in the first position, any number other than four in the second position, any even number other than four in the last position ($=9\cdot 4$), d) a four in the last and second positions and any number other than four in the first position (= 8), e) a four in the last and first positions and any number other than four in the second position (=9), f) a four in the first two positions and any even number other than four in the last position (=4), g) all fours (=1). $8\cdot 9+8\cdot 4+9\cdot 4+8+9+4+1=162$ **A**

13) Law of cosines$\Rightarrow \cos(\alpha+30)=-1/5\Rightarrow\alpha=\cos^{-1}(-1/5)-30$ $\quad \sin\left(\cos^{-1}(-1/5)-30\right)=$
$=\sin\left(\cos^{-1}(-1/5)\right)\cos 30-\cos\left(\cos^{-1}(-1/5)\right)\sin(30)=\left(2\sqrt{6}/5\right)\left(\sqrt{3}/2\right)+(1/5)(1/2)=\left(6\sqrt{2}+1\right)/10$ **D**

14) I) Y = 2, $X = i$ *FALSE*; II) Y = 3, $X=1+i-1-i=0$ *TRUE*; III) see I *FALSE*;
IV) X is 1, $1+i$, i, or 0 *TRUE*; 2 are true **B**

15) The direction of the line is $\mathbf{v} = \begin{bmatrix} -6 \\ 7 \end{bmatrix} - \begin{bmatrix} -1 \\ -3 \end{bmatrix} = \begin{bmatrix} -5 \\ 10 \end{bmatrix}$ $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} -6 \\ 7 \end{bmatrix} + t\begin{bmatrix} -5 \\ 10 \end{bmatrix} = \begin{bmatrix} -6+t \\ 7-2t \end{bmatrix}$ **D**

16) Positive discriminant: $(1+a)^2 - 4a^2 > 0$; $3a+1>0 \Rightarrow a > -1/3$, $-a+1>0 \Rightarrow a<1$ **C**

17) $\left(\sqrt[4]{x} - \sqrt[4]{y}\right)^2 = 1 \Rightarrow \sqrt{x} - 2\sqrt[4]{xy} + \sqrt{y} = 1 \Rightarrow \sqrt[4]{xy} = 1$ $\left(\sqrt[4]{x} - \sqrt[4]{y}\right)^4 = x - 4x^{3/4}y^{1/4} + 6\sqrt{xy} - 4x^{1/4}y^{3/4} + y$
$6\sqrt{xy} = 6(1)^2 = 6$ $-4x^{3/4}y^{1/4} - 4x^{1/4}y^{3/4} = -4\sqrt[4]{xy}\left(\sqrt{x}+\sqrt{y}\right) = -12$ $1 = x+y-12+6 \Rightarrow x+y=7$ **B**

18) $\det\left(\begin{bmatrix} 1-b & 0 & -1 \\ 1 & 2-b & -1 \\ 1 & 1 & -1-b \end{bmatrix}\right) = -b^3 + 2b^2 - b = 0 \Rightarrow b = 0$ or $b=1$; sum = 1 **C**

19) Let θ be the angle opposite the shortest leg. The longest angle bisector bisects the angle opposite the smallest leg. $\theta = \cos^{-1}(4/5)$. The length of the angle bisector, x, is: $x = \dfrac{4}{\cos(\theta/2)} = \dfrac{4}{\sqrt{1/2\,(1+\cos\theta)}} = \dfrac{4}{\sqrt{9/10}} = \dfrac{4\sqrt{10}}{3}$

$a+b+c=17$ **C**

20) $V = 2\pi(60) = 120\pi$ cubic meters after 1 minute. The base is a fixed area of $\pi(4^2) = 16\pi$ square meters.
$h = V/A = 120\pi/16\pi = 7.5$ **A**

21) One path is NNNEEEEEE This is the same as finding the number of distinct combinations of 3 N's and 6 E's. $(9!)/[(6!)(3!)] = (9\cdot 8\cdot 7)/(3\cdot 2) = 84$ **B**

22) 2 different combinations: 1) two \$10's and one \$5; 2) three \$10's. #1 has 3 different combinations and #2 has 1 combination of bills: $\left[\binom{3}{2}\cdot\frac{3}{10}\cdot\frac{2}{9}\cdot\frac{2}{8}\right] + \left[1\cdot\frac{3}{10}\cdot\frac{2}{9}\cdot\frac{1}{8}\right] = \frac{7}{120}$ **C**

23) The first 4 should be known: $1^1 = 1$; $2^2 = 4$; $3^3 = 27$; $4^4 = 256$. The units digit of 5 to any power is 5 and the units digit of 6 to any power is 6. 7, 8, and 9 have patterns:
units digits for 7: $7^1 \to 7$; $7^2 \to 9$; $7^3 \to 3$; $7^4 \to 1$; $7^5 \to 7$ $7^7 = 7^4 \cdot 7^3 \to 3\cdot 1 = 3$
units digits for 8: $8^1 \to 8$; $8^2 \to 4$; $8^3 \to 2$; $8^4 \to 6$; $8^5 \to 8$ $8^8 = 8^4 \cdot 8^4 \to 6^2 \to 6$
units digits for 9: $9^1 \to 9$; $9^2 \to 1$; $9^3 \to 9$; $9^4 \to 1$; $9^5 \to 9$ $9^9 = 9^4 \cdot 9^5 \to 9\cdot 1 = 9$
$1+4+7+6+5+6+3+6+9 = 47 \to 7$ **C**

24) $f(g(f(x))) = \dfrac{1+2x^2+x^4}{x^4} = 9$ $8x^4 - 2x^2 - 1 = 0$; $y = x^2$ $8y^2 - 2y - 1 = 0 \Rightarrow y = 1/2$ or $y = -1/4$ y cannot be negative: $x^2 = 1/2 \Rightarrow x = \pm\sqrt{2}/2$ $a = \sqrt{2}/2$, $b = -\sqrt{2}/2$; $a^2+b^2 = 1$ **A**

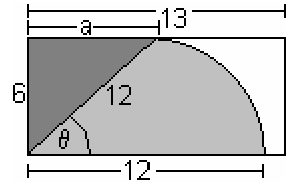
25) We add rows 1 & 2: $-3a+3b=0 \Rightarrow a=b$ Plugging into row 3: $a=b=1$ and row 1: $c=2$;
$a+b+c=-A \Rightarrow A=-4$; $ab+ac+bc=B \Rightarrow B=5$; $abc=-C \Rightarrow C=-2$ $-4+5-2=-1$ **E**

26) $a = 6\sqrt{3}$ $\theta = 90 - \cos^{-1}(6/12) = 30$

Area of triangle = $1/2(6)(6\sqrt{3}) = 18\sqrt{3}$

Area of sector = $1/2(12^2)(\pi/6) = 12\pi$

Total area = $18\sqrt{3} + 12\pi \Rightarrow 18 + 3 + 12 = 33$ **C**



27) A triangle is formed with sides of length 6 miles, 16 miles, and an angle of 60° between. The third side is $d^2 = 6^2 + 16^2 - 2(6)(16)\cos 60^\circ = 36 + 256 - 96 = 196$ $d = \sqrt{196} = 14$ miles **D**

28) $\sum_{n=1}^{2006} n^2 = \dfrac{(2006)(2007)(4013)}{6}$ The last two digits of the sum are only dependent on the last two digits of the numbers of the product: $\dfrac{1}{6}(6)(7)(13) = 7 \cdot 13 = 91$ $1 + 9 = 10$ **B**

29) $R_{eq}(t) = (1/\sin(t) + 1/\tan(t))^{-1} = \sin(t)/(1 + \cos(t)) = \tan(t/2)$ **C**

30) The roots to $x^2 - 54x + 704 = 0$ will be the two values of interest. Using the quadratic equation we attain $x = 22$ or $x = 32$ Difference = 10 **D**