E) NOTA

1) Ten balls are placed in a bag. Four are red, three are blue, two are green, and one is white. Two balls are selected randomly without replacement. What is the probability they are not the same color?

A) 
$$\frac{3}{4}$$
 B)  $\frac{7}{9}$  C)  $\frac{4}{5}$  D)  $\frac{9}{10}$  E) NOTA

- 2) Which of the following conic sections have eccentricities less than or equal to 1?
  - I)  $x^2 y^2 = 4$  II)  $x^2 + y^2 = 4$  III)  $x^2 + 2y^2 = 4$  IV)  $x^2 y = 4$

A) I only B) III only

3) Given:  $x = \sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$  and  $y = \sum_{n=1}^{\infty} \frac{5^{n-1}}{6^{n-1}}$ 

Which of the following are true regarding *x* and *y*?

C) I & II only D) II, III, & IV

n=0 $c$	n=1 o	
A) $x = y$	B) $x > y$	C) $x < y$
D) There is not enough infor	mation E) NOTA	

4) Find the sum of 
$$723_8 + 124_5$$
 in base-9.  
A)  $506_9$  B)  $622_9$  C)  $812_9$  D)  $1141_9$  E) NOTA

5) Find the sum of the zeros of  $g(x) = 2\cos(2x) - 2\sin(3x) + 1$ .  $x \in [0, \pi)$ A)  $\frac{\pi}{2}$  B)  $\pi$  C)  $\frac{3\pi}{2}$  D)  $2\pi$  E) NOTA

- 6) Find the sum of the first twenty terms of the sequence: 1, 3, 7, 15, 31, 63, ... A)  $2^{20} - 22$  B)  $2^{20} - 20$  C)  $2^{21} - 22$  D)  $2^{21} - 20$  E) NOTA
- 7) What is the constant term of the expansion  $\left(\frac{2}{x} x^2 + 1\right)^5$ ? A) -119 B) -19 C) 81 D) 121 E) NOTA

8) Which of the following are NOT vertical asymptotes of the function

$$f(x) = \frac{x^{4} + 3x^{3} - 3x^{2} - 11x - 6}{x^{3} + 5x^{2} + 3x - 9}$$
  
I)  $x = -3$  II)  $x = -1$  III)  $x = 0$  IV)  $x = 1$  V)  $x = 2$   
A) III only B) I & III only C) I & IV only D) II, III, & V E) NOTA

9) Which of the following is a fifth root of  $z = -16\sqrt{2} - 16i\sqrt{2}$ ? (NOTE:  $i = \sqrt{-1}$ ) A)  $\sqrt{2} + i\sqrt{2}$  B)  $\sqrt{2} - i\sqrt{2}$  C)  $-\sqrt{2} - i\sqrt{2}$  D)  $-\sqrt{2} + i\sqrt{2}$  E) NOTA 10) What is the range of the function  $f(x) = \sqrt{1 - e^x}$ ? C) [0,∞) A) (-1,1)B) [0,1] D)  $(-\infty,0]$ E) NOTA 11)  $r = 3 + 3\cos\theta$  is the graph of a: B) 3-Leaved Rose C) Cardioid A) Lemniscate D) Circle E) NOTA 12) How many integers from 100 to 999 inclusive are even and contain at least one digit that is equal to 4? A) 162 B) 190 C) 200 D) 212 E) NOTA

13) In the given figure, what is the exact value of  $sin(\alpha)$ ? (Note: Figure not drawn to scale)



14) Let  $i = \sqrt{-1}$  and  $X = \sum_{k=0}^{Y} i^k$  where *Y* is an integer. How many of the following statements are true?

- I) X is a real number if Y is even.
- II) X = 0 if Y = 4n 1 for all integer values of *n*.
- III) X is never purely imaginary.
- IV) X only takes on four distinct values.

15) Which of the following defines a set of parametric equations describing the line passing through the points (-1,-3) and (-6,7)?

$$\begin{array}{l} \mathbf{A} \\ \mathbf{x}(t) \\ \mathbf{y}(t) \\ \end{bmatrix} = \begin{bmatrix} -1 - 7t \\ -3 + 4t \end{bmatrix} \\ \mathbf{B} \\ \begin{bmatrix} x(t) \\ y(t) \\ \end{bmatrix} = \begin{bmatrix} -1 - 6t \\ -3 + 7t \end{bmatrix} \\ \mathbf{D} \\ \begin{bmatrix} x(t) \\ y(t) \\ \end{bmatrix} = \begin{bmatrix} -6 - t \\ 7 - 3t \end{bmatrix} \\ \mathbf{E} \\ \begin{array}{l} \text{NOTA} \\ \end{bmatrix} \\ \end{array}$$

16) What values of *a* guarantee the quadratic polynomial  $y = a^2 x^2 - (1+a)x + 1$  to have at least two real zeros?

A) 
$$-1 < a < \frac{1}{3}$$
 B)  $a < -1$  or  $a > \frac{1}{3}$  C)  $-\frac{1}{3} < a < 1$  D)  $a < -\frac{1}{3}$  or  $a > 1$  E) NOTA

17) Given:  $\sqrt[4]{x} - \sqrt[4]{y} = 1$  and  $\sqrt{x} + \sqrt{y} = 3$  where x and y are both positive real numbers. Find x + y.

18) Find the sum of the values of b such that the matrix  $B = A - b \cdot I$  has a determinant equal to zero where

 $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & -1 \\ 1 & 1 & -1 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$   $A) -1 \qquad B) 0 \qquad C) 1 \qquad D) 2 \qquad E) \text{ NOTA}$ 19) The length of the longest angle-bisector of a 3-4-5 right triangle can be written as  $\frac{a\sqrt{b}}{c}$  in simplified

form. Find a+b+c.

- A) 10 B) 13 C) 17 D) 21 E) NOTA
- 20) Water is being poured into an empty cylindrical basin of diameter 8 meters and height 10 meters at a rate of  $2\pi$  cubic meters per second. Find the height of the water in meters after 1 minute.

A) 7.5	B) 8.5	C) 9.5
D) The water has already started to c	overflow	E) NOTA

21) How many distinct paths exist from point A to B if you can only move north or east on the grid?



22) A box contains 5 one dollar bills, 2 five dollar bills, and 3 ten dollar bills. Dave digs into the box and grabs three distinct bills at random. Find the probability that Dave picks up a total of at least \$25.

A) 
$$\frac{9}{200}$$
 B)  $\frac{1}{40}$  C)  $\frac{7}{120}$  D)  $\frac{1}{16}$  E) NOTA

23) What is the units digit of  $\sum_{n=1}^{9} n^n$  ?

24) The two real roots of f(g(f(x))) = 9 are denoted as *a* and *b*, where  $f(x) = \frac{1}{x^2}$  and  $g(x) = \frac{1}{1+x}$ . Find  $a^2 + b^2$ .

A) 1 B) 2 C) 3 D) 4 E) NOTA

25) The coordinates of the ordered triple (a,b,c) that satisfies the given system of equations are three zeros of the cubic polynomial f(x) = x<sup>3</sup> + Ax<sup>2</sup> + Bx + C.
Find A + B + C given the system:

-2a+b+c=1

$$a + 2b = 3$$
  
A) -3 B) 0 C) 1 D) 3 E) NOTA

26) A dog is confined in a rectangular pen 6 feet by 13 feet. The dog is on a leash 12 feet long that is tied to a corner of the pen. If the area of which the dog can roam can be expressed as  $a\sqrt{b} + c\pi$  square feet (*a*, *b*, and *c* are real numbers) in simplest form, then find a + b + c. (NOTE: the dog cannot jump out of the pen nor get through the pen and there is no gate to the fence. Also disregard the size of the dog.)

- A) 25 B) 31 C) 33 D) 41 E) NOTA
- 27) Two ships are leaving a dock at the same time. Ship A is traveling at 3 miles/hour at a bearing of 30° while Ship B is traveling at 8 miles/hour at a bearing of 330°. Find the number of miles between the two ships two hours after both ships have left the dock.
  - A) 5 B) 7 C) 10 D) 14 E) NOTA

28) Find the sum of the last two digits of  $\sum_{n=1}^{2006} n^2$ . A) 6 B) 10 C) 12 D) 13 E) NOTA

29) When two resistors,  $R_1$  and  $R_2$ , are combined in parallel the equivalent resistance is given by

 $R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$ . Suppose that we have two resistors that are dependent on time. Which of the

following is equal to the equivalent resistance of the parallel combination of  $R_1(t) = \sin(t)$  and  $R_2(t) = \tan(t)$ .

- A)  $\tan(t)$  B)  $\cot(t)$  C)  $\tan\left(\frac{t}{2}\right)$  D)  $\cot\left(\frac{t}{2}\right)$  E) NOTA
- 30) The sum of two numbers is 54 and their product is 704. What is the difference between these two numbers?
  - A) 4 B) 6 C) 8 D) 10 E) NOTA