1. A
2. D
3. C
4. A
5. C
6. C
7. C
8. B
9. D
10. D
11. A
12. C
13. D
14. C
15. C
16. B
17. B
18. D
19. B
20. C
21. B
22. D
23. B
24. B
25. D
26. C
27. C
28. A
29. A
30. A

1. A The only such triangle is a 3-4-5 right triangle, which has perimeter 12.
2. D Construct a point A at the top of circle D. DA = 8, and OD is a leg of the right triangle FOD. OA is a radius of circle O, so it must be true that OA = DA + OD = 2OF. If we let OF = x, then using the Pythagorean theorem we arrive at the equality $8+\sqrt{(x+8)^2-x^2}=2x$. Solving for x, we get x = 12, and hence the radius of circle O is 24.
3. C Subtract the area of half of circle E and half of circle F from the area of half of circle O. Circles E and F have radii 12, and circle O has radius 24. This yields 144π.
4. A The curve in question is a semicircle of radius 2 centered at the origin. Since the semicircle is defined over the interval [-2, 2], we are computing the area of half of this semicircle, which is π.
5. C Taking the dual of a polyhedron essentially reverses the roles of vertices and faces. A cube has 8 vertices and 6 faces; its dual then has 6 vertices and 8 faces. This describes an octahedron.
6. C We will employ the identity $\cos\theta=\frac{u\cdot v}{\|u\|\cdot\|v\|}$, where u and v are the vectors in question. Since the dot product and the magnitudes are given, we see that $\cos\theta=1/2$. Now we will look at the area formula $A=(1/2)\|u\|\cdot\|v\|\sin\theta$. Since $\cos\theta=1/2$, it follows that $\sin\theta=\sqrt{3}/2$. Substituting the remaining values into the area formula, we get $A=\sqrt{3}/2$.
7. C Construct a right triangle in which one of the acute angles is 2x and the other acute angle is 3x. Then, based on the right triangle definition of sine and cosine, we observe that $\cos(3x)=\sin(2x)$. Therefore, this geometric figure accurately describes the equation we are trying to solve. Now simply notice that 2x + 3x = 90, so x = 18, and the sum of its digits is 9.
8. B If we were to cut the Mobius strip widthwise, a rectangular ribbon of width y would be produced. The total surface area of this object could be found by measuring the length of the ribbon, multiplying it by y, and then doubling the result to account for the two sides of the ribbon. So we are multiplying y by twice the length of the ribbon. But since x forms a complete cycle around the edge of the Mobius strip, it actually traverses the length of the rectangular ribbon twice. Thus the total surface area is xy.
9. D Label the focus of the parabola as F, the vertex of the parabola as V, the second focus of the ellipse as D, the endpoints of the lattice rectum L and R, and finally the endpoints of the major axis of the ellipse as T and B, with T being the endpoint lying "within" the parabola. Since LR = 4, it follows that RF = 2 and FV = DV = 1. DFR is therefore an isosceles right triangle, and so DR = $2\sqrt{2}$. By the definition of an ellipse, since RF + RD = 2 + $2\sqrt{2}$, it must be true that TF + TD = 2 + $2\sqrt{2}$ as well. But TF + TD = 2TF + FD = TF + FD + DB = TB = the length of the major axis. So TB = 2 + $2\sqrt{2}$.
10. D The X, y, x configuration is known as "angle-side-side," and can result in what is called the ambiguous case, where there are two possible values of z. (This is explored in detail in problem 30.)
11. A As the number of sides of a polygon increase while the length of each side remains constant, the area of the polygon will grow larger and approach infinity. However, if as the number of sides of a polygon increase the apothem is held constant, the polygon will approach a circle whose radius has the same measure as the apothem. a_n approaches infinity, while b_n converges. Hence the limit is zero.
12. C The tetrahedron we are looking for will have each edge forming a diagonal of a face of the cube, where opposite edges of the tetrahedron form diagonals on opposite faces of the cube. Each edge of the tetrahedron then has length $\sqrt{2}$, and so each of its faces will have area $\sqrt{3}/2$, and the total surface area will be $2\sqrt{3}$.

13. D By the definition of projection, $\|x\| = \|v\|\cos\theta$ and $\|y\| = \|w\|\cos\theta$. The magnitude of $(x - y)$ will be: $\sqrt{\|w\|^2\cos^2\theta - 2\|v\|\cdot\|w\|\cos^3\theta + \|v\|^2\cos^2\theta}$, or equivalently, $\cos\theta\sqrt{\|w\|^2 - 2\|v\|\cdot\|w\|\cos\theta + \|v\|^2}$, which equals $\cos\theta\|(v-w)\|$. Hence, the ratio of the two quantities will simply be $\cos\theta$.

14. C The maximum value that the function attains on the interval is 6, and the minimum value it attains is 2. Therefore the area under the curve in this interval cannot exceed the max height times the width of the interval, 6(5 – 2) = 18. Similarly, since the function never takes on values less than 2, the area under the curve in this interval must be at least as large as its minimum value times the width of the interval, 2(5 – 2) = 6. This rules out 4 immediately. Now, an area of 6 corresponds to $f(x)$ being a horizontal line of height 2. But $f(x)$ takes on values other than 2, so the area under it cannot equal exactly 6. The only possibility is 8.

15. C The 3-dimensional solid intersection will resemble a rugby ball, with four convex curved faces, four curved edges, and two vertices. When viewed from along the x or y-axis it will appear as a circle. When viewed from along the z-axis, however, it will appear as a square with both diagonals drawn in—a box with an X in it. This figure contains four small triangles and four large triangles (each large triangle made of two adjacent smaller triangles), thus yielding 8 in total.

16. B By unfolding the tetrahedron it becomes clear that the distance along its surface between midpoints of opposite edges is the same as the length of one edge: 1.

17. B A semicircle of radius 1 will fit inside the segment of the sine curve. The sine curve has maximum height 1 but a width of π, which is greater than the width of the semicircle. Furthermore, the segment of the sine curve will in turn fit inside the semicircle of radius $\pi/2$, because the semicircle has the same width as the segment of the sine curve, however the semicircle has a height of $\pi/2$; a greater height than that of the sine curve. The correct ordering is I, III, II.

18. D Since X is the centroid of triangle ABC, AX is twice as long as DX. Therefore AD = 3DX = $\sqrt{13}$. Let DB = x. Then CB = $2x$, and by 30-60-90 right triangle properties, AB = $2x\sqrt{3}$. Using the Pythagorean theorem on triangle ABC, we see that $13 = x^2 + 12x^2$, and solving we get $x = 1$. Now CB = 2 and BE = ½ AB = $\sqrt{3}$. Using Pythagorean theorem again on triangle EBC, we find that CE = $\sqrt{7}$. Finally, using the centroid property once more, we get EX = $\sqrt{7}/3$.

19. B A quadrilateral whose area is one half the product of its diagonals must either be a kite or a rhombus; but of these only a kite has all four vertices on the circle.

20. C Let u = (3, 1) and v = (1, 2). The three possible points for the fourth coordinate of the parallelogram can be represented by $(u + v)$, $(u - v)$, and $(v - u)$: (4, 3), (2, -1), and (-2, 1), respectively. The area of the triangle defined by these points is 10.

21. B The width of the rectangle in question is defined as the major axis, or 2(4) = 8, and its height is defined as the minor axis, or 2(5) = 10. (8)(10) = 80.

22. D Triangle AQB must be a right triangle with right angle Q because one of its sides, AB, is a diameter of a circumscribed circle. A and B both lie on circle Q, so QA and QB are both radii of circle Q, and hence must be equal. Therefore triangle AQB is an isosceles right triangle, and it can be seen that the ratio of AQ to AO is $\sqrt{2}$, and hence the ratio of the area of circle Q to that of circle O is 2.

23. B Notice that r is an altitude of triangle JMK, whose base has measure s. Therefore the area of triangle JMK is ½ rs, and the area of JKLM, which is twice as large as JMK, is rs. Now notice that the length of the long diagonal of JKLM is $2R$, while the short diagonal is defined as d. Therefore the area of JKLM is ½ $(2R)(d) = Rd$. Hence, $72 = rRds = (rs)(Rd)$ = (area of JKLM)^2. So the area of JKLM is $6\sqrt{2}$.

24. B A regular five-pointed star can be represented by a regular pentagon each of whose sides form the base of an isosceles triangle such that the base angles of each triangle are supplementary to interior

angles of the pentagon. The interior angle of the pentagon has measure 108, so the base angles of the isosceles triangles must each have measure 180 – 108 = 72. The interior angle of the point of one of these triangles then must have measure 180 – 2(72) = 36.

25. D Extend side AD to a point E on side CB. Triangle DEC will now be a 45-45-90 right triangle, so DE and CE will both have measure $3\sqrt{2}$. Triangle AEB will be a 30-60-90 right triangle with AE = 6 + $3\sqrt{2}$. Then EB = $(6+3\sqrt{2})/\sqrt{3} = 2\sqrt{3}+\sqrt{6}$, and finally CB = CE + EB = $2\sqrt{3}+3\sqrt{2}+\sqrt{6}$.

26. C The figure being described is an ellipse, and the distance d is equivalent to the length of the major axis of the ellipse. Let $a = d/2$, the "major radius" of the ellipse. Let $c = (3/10)d$ = half the distance between A and B, the "focal radius" of the ellipse. To find b, the "minor radius" of the ellipse, we will use the Pythagorean identity for ellipses, $a^2 = b^2 + c^2$. This yields $b = (2/5)d$. The area of the ellipse is then $\pi(2/5)d\cdot(1/2)d = d^2\pi/5$.

27. C Since BD is an angle bisector, the property holds that AB/AD = BC/DC. So 6/AD = 4/(5 – AD). Solving for AD we get AD = 3.

28. A Since the volume of a cone is one-third the volume of a cylinder with the same base and height, in order for their volumes to be equivalent the cone must be three times as tall as the cylinder it shares a base with.

29. A The longer side of the golden rectangle with shortest side x must have length ϕx, which will also be the length of the shorter side of the second golden face of the prism. Hence the longer side of this second face will have length $\phi^2 x$. Therefore the non-golden face must have dimensions $\phi^2 x$ by x, and area $\phi^2 x^2$.

30. A The given information is an example of the "angle-side-side" ambiguous case, in which angle C could have two different valid values—one greater than 90 and one less than 90. The obtuse option will yield a smaller length for the unknown side, AC, and so this is the scenario we will examine. Drop an altitude down from B, and extend segment AC past point C so that it intersects D. Now triangle BDA is a 30-60-90 right triangle with hypotenuse 12. AD is therefore 6, BD is $6\sqrt{3}$, and the area of triangle BDA is $18\sqrt{3}$. Then, by the Pythagorean theorem, CD = $\sqrt{11^2-(6\sqrt{3})^2} = \sqrt{13}$, and the area of triangle BDC is $3\sqrt{39}$. Finally, the area of triangle ABC = area of BDA – area of BDC = $18\sqrt{3}-3\sqrt{39}$.