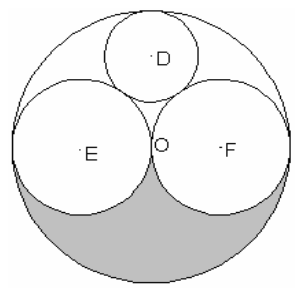
1. In a right triangle with integral side lengths, one of the sides has length 4. What is the perimeter of this triangle?

A. 12 B. 28 C. 49

D. There is more than one such triangle E. NOTA

2. In the figure, circles E and F are congruent.
If circle D has radius 8, find the radius of circle O.



A. 6
B. 8
C. 12
D. 24
E. NOTA

3. Referring to the same figure as the in the previous question, find the area of the shaded region.

A. 72π B. 108π C. 144π D. 180π E. NOTA

4. Find the area under the curve $f(x) = \sqrt{4 - x^2}$ from $x = 0$ to $x = 2$.

A. π B. 2π C. 4π D. 8π E. NOTA

5. A *dual polyhedron* is a polyhedron constructed by placing a point in the center of each face of an existing polyhedron, and then connecting with a straight line any two such points that reside on adjacent faces of the original polyhedron. Based on this definition, what is the dual polyhedron of a cube?

A. tetrahedron B. cube C. octahedron D. dodecahedron E. NOTA

6. Determine the area of the triangle formed by connecting the tips of two vectors with respective magnitudes 1 and 2, and whose dot product is 1.

A. $\frac{\sqrt{3}}{4}$ B. $\frac{1}{2}$ C. $\frac{\sqrt{3}}{2}$ D. 1 E. NOTA

7. Find the sum of the digits of x if $\cos(3x) = \sin(2x)$ and x lies between 0° and 60°.

A. 5 B. 6 C. 9 D. 11 E. NOTA

8. A point p is placed on one edge of a Möbius strip with uniform width y. A path starting at p is drawn along the edge of the Möbius strip until it meets back at p again. The length of this path is x. Find the total surface area of the Möbius strip.

A. $\frac{xy}{2}$ B. xy C. $2xy$
D. Cannot be determined E. NOTA

9. An ellipse and a parabola are defined as follows. The focus of the parabola coincides with one focus of the ellipse, the vertex of the parabola coincides with the center of the ellipse, and the ellipse passes through both endpoints of the latus rectum of the parabola, which has length 4. Find the length of the major axis of the ellipse.

A. $2\sqrt{2}$ B. $1+\sqrt{2}$ C. $2+\sqrt{2}$ D. $2+2\sqrt{2}$ E. NOTA

10. Triangle XYZ has sides x, y, and z, with x opposite X, y opposite Y, and z opposite Z. Which of the following sets of measures is not necessarily sufficient information to determine the area of XYZ?

A. x, y, and z B. X, Y, and z C. x, Y, and z D. X, y, and x E. NOTA

11. Let a_n = the area of a regular n-gon with side length k, and let b_n = the area of a regular n-gon with apothem length h. Evaluate $\lim_{n\to\infty} \frac{b_n}{a_n}$.

A. 0 B. $\frac{h}{k}$ C. Does not exist
D. Not enough information E. NOTA

12. Find the surface area of the largest regular tetrahedron that can fit inside a cube of side length 1.

A. $\frac{\sqrt{3}}{2}$ B. $\sqrt{3}$ C. $2\sqrt{3}$ D. $8\sqrt{3}-12$ E. NOTA

13. Consider two vectors, $\bar{v}$ and $\bar{w}$. Define $\bar{x}$ as the projection of $\bar{v}$ onto $\bar{w}$, and $\bar{y}$ as the projection of $\bar{w}$ onto $\bar{v}$. Find the ratio of the magnitude of ($\bar{x} - \bar{y}$) to the magnitude of ($\bar{v} - \bar{w}$), in terms of theta, the angle between $\bar{v}$ and $\bar{w}$.

A. $1/\sin(\theta)$ B. $\sin(\theta)$ C. $1/\cos(\theta)$ D. $\cos(\theta)$ E. NOTA

14. A continuous function $f(x)$ is defined on the closed interval [2, 5], with endpoint values $f(2) = 4$ and $f(5) = 2$. Furthermore, the relative extrema of $f(x)$ on [2, 5] consist of a relative maximum at (3, 6) and a relative minimum at (4, 3). Which of the following could be a possible value for the area under the curve $f(x)$ over the interval [2, 5]?

A. 4   B. 6   C. 8

D. More than one of these values could be valid   E. NOTA

15. A right circular cylinder of radius r and infinite height is constructed along the entire x-axis such that any perpendicular cross section of the cylinder will be a circle whose center lies on the x-axis. A congruent cylinder is also constructed similarly along the y-axis. The three dimensional solid intersection of these two cylinders, when viewed from above looking down the z-axis, will appear as a two dimensional planar figure. How many triangles will this planar figure contain?

A. 0   B. 4   C. 8   D. 16   E. NOTA

16. An ant standing on the center of one edge of a regular tetrahedron of side length 1 wishes to crawl along the surface of the figure to the center of an edge of the tetrahedron that is skew to the edge he is currently standing on. What is the shortest distance the ant can crawl to achieve his goal?

A. $\frac{1}{2}$ B. 1   C. $\sqrt{2}$ D. $\sqrt{3}$ E. NOTA

17. Put the following in order from least to greatest:

I. The area of a semicircle of radius 1

II. The area of a semicircle of radius $\frac{\pi}{2}$

III. The area under the curve $y = \sin(x)$ from $x = 0$ to $x = \pi$.

A. I, II, III   B. I, III, II   C. III, I, II   D. II, III, I   E. NOTA

18. In triangle ABC, angle A has measure 30° and angle C has measure 60°. A median is drawn from A to a point D on BC, and another median is drawn from C to a point E on AB. The medians intersect at a point X. If DX is $\frac{\sqrt{13}}{3}$, then find EX.

A. 1   B. 2   C. $\sqrt{7}$ D. $\frac{\sqrt{7}}{3}$ E. NOTA

19. A quadrilateral is inscribed in a circle such that the quadrilateral's area equals half the product of its diagonals, and all four of its vertices lie on the circle. What is the most specific classification of such a quadrilateral?

A. trapezoid B. kite C. rhombus D. square E. NOTA

20. Three of the four vertices of a parallelogram are (0, 0), (3, 1), and (1, 2). Find the area of the triangle whose vertices are the three possible coordinates of the fourth vertex of the parallelogram.

A. 2.5 B. 5 C. 10 D. 20 E. NOTA

21. Consider the following hyperbola: $25(x-2)^2 - 16(y-3)^2 = 400$.
Find the area of the rectangle that intersects this hyperbola exactly twice and whose diagonals lie along the hyperbola's asymptotes.

A. 20 B. 80 C. 100 D. 400 E. NOTA

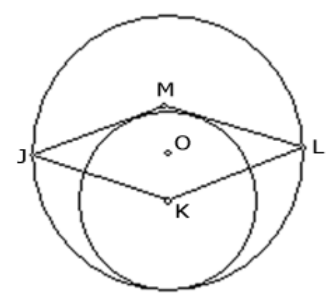
22. Line segment AB is a diameter of circle O and a chord of circle Q. The center of circle Q lies on circle O. Find the ratio of the area of circle Q to the area of circle O.

A. 1 B. $\sqrt{2}$ C. $\sqrt{3}$ D. 2 E. NOTA

23. Points J and L of rhombus JKLM form endpoints of a diameter of circle O. A circle with center K is tangent to circle O as well as to sides LM and MJ of the rhombus.

Consider the radius r of circle K,
the radius R of circle O,
the shorter diagonal d of the rhombus,
and the side s of the rhombus.

If $rRds = 72$, then what is the area of rhombus JKLM?



A. 6 B. $6\sqrt{2}$ C. 36 D. 72 E. NOTA

24. Find the measure of the interior angle of one point of a regular five-pointed star.

A. 27° B. 36° C. 54° D. 72° E. NOTA

25. In concave quadrilateral ABCD, the interior angles measure as follows:
A = 30°, B = 60°, C = 45°, and D = 225°.
If AD = DC = 6, then what is the length of BC?

A. $2\sqrt{3}+\sqrt{6}$ B. $3\sqrt{2}+\sqrt{6}$ C. $4\sqrt{3}+3\sqrt{2}$

D. $2\sqrt{3}+3\sqrt{2}+\sqrt{6}$ E. NOTA

26. Consider a curve in the plane where the sum of the distances of any point on the curve to fixed points A and B is *d*. If the distance between A and B is $\frac{3}{5}d$, find the area of the region bounded by this curve.

A. $\frac{3d^2\pi}{25}$ B. $\frac{9d^2\pi}{25}$ C. $\frac{d^2\pi}{5}$ D. $\frac{3d^2\pi}{5}$ E. NOTA

27. In triangle ABC, AB = 6, BC = 4, and AC = 5. An angle bisector is drawn from B to a point D on side AC. Find AD.

A. 1 B. 2 C. 3 D. 4 E. NOTA

28. A right circular cone shares its base with one base of a right circular cylinder. If the volume of the cone and the volume of the cylinder are equal, what is the ratio of the height of the cylinder to the height of the cone?

A. 1/3 B. 1/9 C. 3 D. 9 E. NOTA

29. Two of the faces of a rectangular prism are non-congruent golden rectangles, and the shortest side of the smaller of these two rectangles has length *x*. Find the area of one of the non-golden rectangular sides of the prism, in terms of *x* and the golden ratio, ϕ.

A. ϕ^2x^2 B. ϕ^2x C. ϕx^2 D. ϕ^3x^2 E. NOTA

30. In triangle ABC, angle A = 60°, AB = 12, and BC = 11. Find the smaller of the two possible areas of this triangle.

A. $18\sqrt{3}-3\sqrt{39}$ B. $18\sqrt{3}+3\sqrt{39}$ C. $36\sqrt{3}-6\sqrt{39}$

D. $36\sqrt{3}+6\sqrt{39}$ E. NOTA