1. C
2. B
3. D
4. C
5. D
6. D
7. A
8. D
9. E
10. B
11. A
12. A
13. D
14. C
15. B
16. D
17. B
18. C
19. D
20. E $\frac{19}{20}$
21. D
22. C
23. B
24. A
25. C
26. C
27. B
28. E 0
29. C
30. B

1. C – The probability of winning neither is the probability of not winning a door prize times the probability of not winning a trophy. $\left(1-\frac{1}{10}\right)\left(1-\frac{1}{3}\right)=\frac{9}{10}\frac{2}{3}=\frac{3}{5}$

2. B – We need 2 sixes and 10 non-sixes. There are 12 choose 2 = 66 ways of ordering the sixes among the 12 rolls.

3. D - $2x^2-17x+30=(2x-5)(x-6)$ which has solutions 2.5 and 6. Since the parabola is open up, the probability is (6-2.5)/10 = 7/20.

4. C – The first card can be anything, the second can be any one of 48 out of the remaining 51 cards. 48/51 = 16/17.

5. D – P(F|E) = P(EF)/P(E) = P(E|F)P(F)/P(E) = 0.

6. D – There are 5 choose 2 ways of picking the boys and 8 choose 2 ways of picking 2 students. 10/28 = 5/14.

7. A – Count the ways, neither Arne or Bob can win. All 6 ways that Charlie wins are good and if Dave wins the good ways are: DCAB, DCBA and DACB. A total of 9 ways.

8. D – There are 11 integers in the set including 0. To get a negative product, we must select a positive (5/11) and a negative (5/11) and they can occur in either order. 2(5/11)(5/11) = 50/121.

9. E (1/2) – There are 33 multiples of 3 and 25 multiples of 4. However, these share the 8 multiples of 12. So, there are 33+25-8=50 multiples of 3 or 4. 50/100 = ½.

10. B – The last four flips must be THHH which occurs with probability 1/16. Of the 16 ways in which the first 4 flips occur, HHHH, HHHT and THHH would end the game prematurely. The probability is then (13/16)(1/16).

11. A – Let *n* be the number of each color of marble. $\frac{n}{2n}\left(\frac{n}{2n-1}\right)$ is the probability of first drawing a blue and then a yellow marble. Since order does not matter, multiple this by 2, set equal to 8/17 and solve for *n*.

12. A – There are 6! ways of permuting the numbers 1 through 6 and thus 6! ways of getting one of each. The are 6^6 ways of rolling a die six times. $\frac{6\cdot5\cdot4\cdot3\cdot2}{6\cdot6\cdot6\cdot6\cdot6\cdot6}=\frac{5}{324}$

13. D – If the third vertex is on the circumference of the circle that has (0,2) and (4,2) as a diameter then the angle at that vertex will be right. Anywhere inside the circle and the angle will be obtuse. Note that the other angles cannot be obtuse because the point must lie in the square and that the above circle is contained in the given square. The probability is then the area of the circle, π, divided by the area of the square, 4.

14. C – There are 31 combinations that have at least one girl. Any one of the 16 combinations of the first 4 kids will work as long as the last is a girl. The answer is then 16/31.

15. B – Of the 9 choose 2 = 36 ways to pick the integers; the number 2 goes with (3,5,7,9), 3 goes with (4,5,6,7,8,10), 4 with (5,7,9), 5 with (6,7,8,9), 6 with (7), 7 with (8,9,10), 8 with (9) and 9 with (10). 22/36 = 11/18.

16. D – Suzanne wins in the 1st round with probability (1/2)(1/3)=1/6. No one wins in any given round with probability (1/2)(2/3)=1/3. So the probability that Suzanne wins is: (1/6)[1/3 + 1/9 + 1/27 +…] = (1/6)[1/(1-1/3)]=(1/6)(3/2)=1/4.

17. B $P(B)=\frac{P(AB)}{P(A\mid B)}=\frac{P(B\mid A)P(A)}{P(A\mid B)}=\frac{0.6(0.4)}{0.3}=0.8$

18. C – If Janne wins the first game, there are 15 choose 10 ways the other games could be ordered. In all there are 16 choose 11 ways of ordering the games. Dividing, the answer is 11/16.

19. D – This is a classic problem. The Mu that came to the door could be any one of the three on the block. Of those three, two have another Mu in the house (2/3).

20. E (19/20) – A^c union B is all the possibilities except A intersection B^c, so the probability is 1 – (1/4 – 1/5) = 19/20.

21. D – Let G_1 and G_2 be the event of drawing a green M&M out of bowl 1 or 2 respectively and R_1 the event of a red from bowl 1.

$$P(G_1 \mid G_2) = \frac{P(G_1 G_2)}{P(G_2)} = \frac{P(G_2 \mid G_1)P(G_1)}{P(G_2 \mid G_1)P(G_1) + P(G_2 \mid R_1)P(R_1)}$$

$$= \frac{\frac{8}{12}\left(\frac{5}{11}\right)}{\frac{8}{12}\left(\frac{5}{11}\right) + \frac{7}{12}\left(\frac{6}{11}\right)} = \frac{\frac{40}{132}}{\frac{82}{132}} = \frac{40}{82} = \frac{20}{41}$$

22. C – The number of squares that the diagonal line will pass through is m + n – gcd(m,n) where m is the number of rows and n the number of columns and gcd is the greatest common divisor. S0, 8+12-4 = 16 squares with lines. 16/96 = 1/6.

23. B – Probability of at least one is 1 – P(none) =

$$1 - \frac{\binom{9}{4}\binom{3}{0}}{\binom{12}{4}} = 1 - \frac{9 \cdot 8 \cdot 7 \cdot 6}{12 \cdot 11 \cdot 10 \cdot 9} = 1 - \frac{14}{55} = \frac{41}{55}$$

24. A – John can either win the first two games, or one of the first two games and the third.

$$\frac{3}{5}\left(\frac{3}{5}\right) + 2\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right) = \frac{81}{125}$$

25. C – The total number of paths A to B is $\binom{11}{5}$ and going through C is $\binom{6}{2}\binom{5}{3}$. Dividing these, the answer is 25/77.

26. C – To be divisible by 11, the sum of the digits in the even places (10, 1000, etc) must differ from the sum in the odd places by a multiple of 11. Because of the numbers chosen, the two sums must be equal and there must be two 7's in the even positions and two 7's in the odd positions. There are 4 choose 2 ways of placing the 7's in the even spots and 4 choose 2 ways of placing the 7's in the odd spots. Total there are 8 choose 4 ways of placing the 7's.

$$\frac{\binom{4}{2}\binom{4}{2}}{\binom{8}{4}} = \frac{6 \cdot 6 \cdot 4 \cdot 3 \cdot 2}{8 \cdot 7 \cdot 6 \cdot 5} = \frac{18}{35}$$

27. B – One could set this up as the classic "Hat check" problem but it is easier to just count the ways. Assume (*wlog*) that Ann's order is Ace, King, Queen. Of the 6 permutations of Bobby's hand only King, Ace, Queen and Queen, Ace, King do not have a match. The probability is then 4/6 = 2/3.

28. E (0) – Not including leading zeroes (same answer anyway), there are $9 \cdot 10 = 90$, 3-digit palindromes since the 3rd digit must match the first. There is the same number of 4-digit palindromes as the last two digits are determined by the first two.

29. C – Any one of the three games preferred by two players gives the same result so the probability is:

$$\frac{3\binom{5}{1}\binom{5}{1}\binom{5}{2}}{\binom{15}{4}} = \frac{3 \cdot 5 \cdot 5 \cdot 10}{15 \cdot 14 \cdot 13 \cdot 12} = \frac{50}{91}$$

30. B – Take a right triangle and draw the perpendicular bisectors to the two legs. Anywhere in the shaded region is closer to the right angle and is ½ the area.