



Mu Alpha Theta
2006 National Convention

Trigonometry
Alpha Division

1. What type of conic section is described by the parametric equations, $x = \sin^2 t$ and $y = 5\cos t$?
A. Hyperbola B. Parabola C. Ellipse D. Circle E. NOTA
2. Given that $\cos \theta = \frac{2}{3}$, what is $\cos 5\theta$ given that $0 < \theta < \frac{\pi}{2}$?
A. $-\frac{115}{243}$ B. $-\frac{116}{243}$ C. $-\frac{13}{27}$ D. $-\frac{118}{243}$ E. NOTA
3. What is the period of the function $f(x) = \frac{\sin 16x + \sin 7x}{\cos 16x + \cos 7x}$?
A. $\frac{\pi}{12}$ B. $\frac{\pi}{11}$ C. $\frac{2\pi}{23}$ D. $\frac{\pi}{10}$ E. NOTA
4. The function $f(x) = \cos(x - \frac{\pi}{2})$, where $x \in \left[-\pi, \frac{3\pi}{2}\right]$, passes through how many quadrants?
A. 1 B. 2 C. 3 D. 4 E. NOTA
5. If $\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} = 16$, and x is on the interval $\left[\frac{\pi}{2}, \frac{3\pi}{4}\right]$, find $\sin x + \cos x$.
A. $-\frac{\sqrt{3}}{2}$ B. $\frac{\sqrt{3}}{2}$ C. $-\frac{\sqrt{2}}{2}$ D. $\frac{\sqrt{2}}{2}$ E. NOTA
6. Evaluate: $\lim_{x \rightarrow \pi} \frac{e^{3ix} - e^{-3ix}}{2i}$
A. -1 B. 0 C. 1 D. DNE E. NOTA
7. If $\arcsin \frac{1}{1-x^3} = \theta$, find $\csc \frac{\theta}{2}$ in terms of x , given that θ is a quadrant II angle.
A. $\frac{2-2x^3}{1-x\sqrt{x^3-2x-x^3}}$ B. $\sqrt{\frac{2-2x^2}{1-x\sqrt{x^2-2x-x^2}}}$ C. $\sqrt{\frac{2-2x^3}{1-x\sqrt{x^4-2x-x^3}}}$ D. $\frac{1}{2}$ E. NOTA



8. Evaluate: $\prod_{r=1}^5 e^{i\pi r^3}$
- A. 0 B. -1 C. 1 D. $-\frac{1}{2}$ E. NOTA
9. Rewrite $\sin 3x$ in the form $a \sin^3 x + b \sin x$ where $\{a, b\} \in \mathbb{Z}$, find $567a + 384b$.
- A. -1 B. -933 C. -1116 D. -1299 E. NOTA
10. Which of the following angles is co-terminal to 3864° ?
- A. 14304° B. 2704° C. 1896° D. 1356° E. NOTA
11. What is the range of $\text{Arc csc } x$?
- A. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ B. $[-1, 1]$ C. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ D. $[0, \pi]$ E. NOTA
12. Find the eccentricity of the polar conic $r = \frac{1\left(\frac{3}{4}\right)}{1 + \frac{1}{2}\cos\theta}$.
- A. $\frac{3}{4}$ B. $\frac{1}{2}$ C. $\frac{1}{\sqrt{2}}$ D. 1 E. NOTA
13. Determine the smallest possible angle of rotation needed to eliminate the xy term from the equation,
 $2x^2 + y^2 + \sqrt{2}x + 2y + \sqrt{3}xy = 0$.
- A. 120° B. 60° C. 30° D. 15° E. NOTA
14. What is the range of the function $f(x) = \csc x \tan x \sin x$?
- A. $(-\infty, \infty)$ B. $(0, \infty)$ C. $[0, \pi]$ D. $(-\infty, 0)$ E. NOTA
15. Evaluate: $\cos 36^\circ + \sin^2 18^\circ$
- A. $\frac{5+\sqrt{5}}{8}$ B. $\frac{1}{4}$ C. $\frac{4+\sqrt{3}}{8}$ D. $\frac{3+\sqrt{2}}{4}$ E. NOTA



16. Which of the following polar coordinates are equivalent to the rectangular coordinates $(\sqrt{6} - \sqrt{2}, \sqrt{6} + \sqrt{2})$?

- A. $\left(4, \frac{\pi}{12}\right)$ B. $\left(1, \frac{\pi}{12}\right)$ C. $\left(1, \frac{7\pi}{12}\right)$ D. $\left(4, \frac{5\pi}{12}\right)$ E. NOTA

17. Evaluate: $\sum_{n=0}^{2005} cis(n^2\pi)$, where $cis\theta = \cos\theta + i\sin\theta$

- A. -1 B. 0 C. 1 D. π E. NOTA

18. Find the sum of the solutions of $\cos x + \sin 2x = 0$ on the interval $(0, 2\pi]$.

- A. 2π B. 3π C. 4π D. 5π E. NOTA

19. What is the sine of the angle between the vectors $3i - 2j$ and $i + j$?

- A. $\frac{1}{\sqrt{26}}$ B. $\frac{2\sqrt{3}}{\sqrt{26}}$ C. $\frac{1}{\sqrt{13}}$ D. $\frac{5}{\sqrt{26}}$ E. NOTA

20. Which of the following is equivalent to $\cos 75^\circ$?

- A. $\frac{\sqrt{6}-\sqrt{2}}{4}$ B. $\frac{1}{2}$ C. $\frac{\sqrt{3}}{2}$ D. $\frac{\sqrt{6}+\sqrt{2}}{4}$ E. NOTA

21. Given that $\cos\theta = -\frac{2}{3}$ and θ lies in quadrant II, what is $\tan\left(\frac{\theta}{2}\right)$?

- A. $\frac{3\sqrt{5}}{2}$ B. $\frac{1}{\sqrt{5}}$ C. $\frac{\sqrt{5}}{3}$ D. $\frac{\sqrt{5}}{2}$ E. NOTA

22. In triangle ABC, if $AB = 7$, $BC = 5$, and $\sin B = \frac{1}{5}$, what is the product of all possible lengths for AC?

- A. $2\sqrt{193}$ B. $3\sqrt{57}$ C. 35 D. $\sqrt{773}$ E. NOTA

23. Find the sum of the solutions of $\cos\frac{x}{20} = \sin\frac{x}{10}$ on the interval $(0, 20\pi]$.

- A. 2π B. 15π C. 30π D. 60π E. NOTA



24. When $\sqrt{\sec x + 2\sqrt{\sec x + 2\sqrt{\sec x}}}$ is expressed in the form $a + b\sqrt{c + \sec x}$ where $a, b, c \in \mathbb{C}$, what is a possible value of $a + b + c$?

- A. 17 B. 15 C. 5 D. 1 E. NOTA

25. If the equation $\cos^{2006} x = \cos 2006x$ has N solutions on the interval $(0, 8\pi)$, find the sum of the digits of N .

- A. 8 B. 9 C. 10 D. 11 E. NOTA

26. Evaluate: $\left(\frac{3}{2} + \frac{\sqrt{3}}{2}i\right)^9$

- A. $-81\sqrt{3}i$ B. $19683i$ C. $\frac{-81\sqrt{3}}{2} + \frac{243}{2}i$ D. $\frac{81\sqrt{3}}{2} - \frac{243}{2}i$ E. NOTA

27. What is the approximation of $\sin 3$ when using a 5th degree Taylor polynomial?

- A. $\frac{1}{2}$ B. $\frac{21}{40}$ C. $\frac{11}{20}$ D. $\frac{23}{40}$ E. NOTA

28. Evaluate: $\lim_{\theta \rightarrow 2006\pi} \sqrt{\cos \frac{\theta}{4}}$

- A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. 0 D. DNE E. NOTA

29. Simplify: $\frac{(3+i)(7-2i)(9-3i)}{(5-3i)}$

- A. $\frac{643}{17} + \frac{167}{17}i$ B. $210 + 60i$ C. $210 - 60i$ D. $\frac{615}{17} + \frac{165}{17}i$ E. NOTA

30. The graph of $\frac{\cos(\cos(\cos(\dots\cos(x))))})}{\csc(\csc(\csc(\dots\csc(x))))})}$ can best be described as being symmetrical about which of the following?

- A. Y axis B. X axis C. $y = x$ D. The Origin E. NOTA