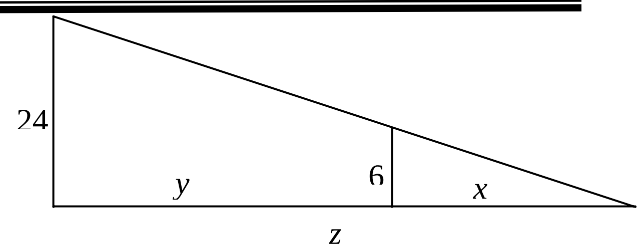
1. C.
2. C.
3. C.
4. D.
5. B.
6. D.
7. C.
8. A.
9. A.
10. D.
11. E.
12. C.
13. A.
14. B.
15. A.
16. D.
17. B.
18. E.
19. C.
20. B.
21. C.
22. D.
23. B.
24. C.
25. C.
26. A.
27. E
28. B.
29. A
30. C.



1. C. $\frac{dx}{dt}=2$; $\frac{6}{24}=\frac{x}{z}\Rightarrow 4\frac{dx}{dt}=\frac{dz}{dt}\Rightarrow\frac{dz}{dt}=8$

   Since $x+y=z\Rightarrow\frac{dx}{dt}+\frac{dy}{dt}=\frac{dz}{dt}\therefore\frac{dy}{dt}=6$

2. C. If you "unroll" the cylinder in your mind, the mark is the hypotenuse of a right triangle with sides 15 and 15π. Therefore, the hypotenuse is the square root of the sum of the squares of those two sides.

3. C. $\lim\limits_{x\to 0}\frac{\tan 2x}{x\cos x} = \frac{2}{1}\lim\limits_{2x\to 0}\left(\frac{\sin 2x}{2x}\times\frac{1}{\cos 2x\times\cos x}\right)=2\times 1\times 1=2$

4. D. $y=\tan\left(x+y\right)\Rightarrow \frac{dy}{dx} = \sec^2\left(x+y\right)\times\left(1+\frac{dy}{dx}\right)\Rightarrow\left(1-\sec^2\left(x+y\right)\right)\frac{dy}{dx}=\sec^2\left(x+y\right)$ but

   $\left(1-\sec^2\left(x+y\right)\right)=-\tan^2\left(x+y\right)$, so $\frac{dy}{dx} = \frac{\sec^2\left(x+y\right)}{-\tan^2\left(x+y\right)} = -\csc^2\left(x+y\right)$

5. B. $\int_a^b f\left(x_k\right)\times\Delta x=\lim\limits_{n\to\infty}\sum_{k=1}^{n}\left(f\left(x_k\right)\times\Delta x\right)$ $\Delta x=\frac{b-a}{n}$, $x_k=a+k\times\Delta x$ so a = 2, b = 5, and

   $f\left(x\right)=\frac{1}{x}$.

6. D. $\int_0^{\pi}\sin^2 xdx = \int_0^{\pi}\left(\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)d\theta=\frac{\theta}{2}-\frac{1}{4}\sin 2\theta\Big|_0^{\pi}=\frac{\pi}{2}$

7. C. There is a "hole" down the middle of the solid so the disk method doesn't fit. The washer method requires an inside and an outside radius but they are both determined by the same relation (since the inverse of $f\left(x\right)=4x-x^3$ is not a function) it is extremely "inconvenient" to solve for x in terms of y. The very same criticisms can be used for option D. That leaves us with the shell method which handles the problem quite nicely.

8. A. $\int_2^5\left|2x-9\right|dx=\int_2^{4.5}\left(9-2x\right)dx+\int_{4.5}^5\left(2x-9\right)dx=6.25+.25=6.5$

9. A. since $\int_{0.081}^{1.372}\sec^2 xdx = 4.883$, $\int_{0.081}^{1.372}\tan^2 xdx=\int_{0.081}^{1.372}\left(\sec^2-1\right)xdx=\int_{0.081}^{1.372}\sec^2 xdx-\int_{0.081}^{1.372}1dx$

   = 4.883 – (1.372 – 0.081) = 3.592

10. D. $y=x^x$. $\ln y=x\ln x$, $\frac{dy}{y}=x\cdot\frac{1}{x}+1\cdot\ln x$., $dy=y\cdot\left(1+\ln x\right)=x^x+x^x\ln x$

11. E. $\lim\limits_{x\to 2}\frac{\sqrt{2}-\sqrt{x}}{2-x} = \lim\limits_{x\to 2}\frac{\sqrt{2}-\sqrt{x}}{\left(\sqrt{2}-\sqrt{x}\right)\left(\sqrt{2}+\sqrt{x}\right)} = \lim\limits_{x\to 2}\frac{1}{\left(\sqrt{2}+\sqrt{x}\right)}=\frac{1}{2\sqrt{2}}$

12. C. $A=\int_0^4\left[\left(4x-x^2\right)-\left(x^3-2x^2-8x\right)\right]dx=\frac{160}{3}$;

$$\bar{y}=\frac{3}{160}\int_0^4\left[\frac{\left[\left(4x-x^2\right)+\left(x^3-2x^2-8x\right)\right]}{2}\times\left[\left(4x-x^2\right)-\left(x^3-2x^2-8x\right)\right]\right]dx,=\frac{-872}{175}$$

13. A. $\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}=\left(5t^4-4t^3\right)\cdot\frac{1}{3t^2}=\frac{5t^2-4t}{3}$; $\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\frac{dx}{dt}}=\frac{\frac{10t-4}{3}}{3t^2}=\frac{10t-4}{9t^2}$

14. B. $\int_5^9\sqrt{-x^2+10x-9}\,dx=\int_5^9\sqrt{16-\left(x-5\right)^2}\,dx$ This is an integral that gives the area of the quarter circle (radius = 4) above the x axis and to the right of x = 5. Therefore, 4π

15. A. The graph is a three leafed rose. The area enclosed by the graph can be represented by
$$A=6\times\frac{1}{2}\int_0^{\frac{\pi}{6}}\left(3\cos\left(3\theta\right)\right)^2d\theta=6\times\frac{9}{2}\int_0^{\frac{\pi}{6}}\left(\frac{1}{2}+\frac{1}{2}\sin\left(3\theta\right)\right)d\theta=\frac{9\pi}{4}$$

16. D. The nth term test can only be used to show that a series diverges when $\lim_{n\to\infty}a_n\neq 0$. C is no good because you were not told that $\{a_n\}$ was a POSITIVE real valued sequence.

17. B. $\sum_{x=3}^{\infty}\frac{5}{x^2+5x}=\sum_{x=3}^{\infty}\left(\frac{1}{x}-\frac{1}{x+5}\right)=\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}=\frac{153}{140}$

18. C. $\lim_{x\to\infty}\frac{\sqrt{x^4-127x^2}}{\sqrt[3]{x^6+95x^5}}=1$ therefore, by the nth term test, the series diverges.

19. C. $\frac{dy}{dx}=2\tan\sqrt{x}\cdot\frac{d}{dx}\left[\tan\sqrt{x}\right]=2\tan\sqrt{x}\cdot\sec^2\sqrt{x}\cdot\frac{d}{dx}\left[\sqrt{x}\right]=2\tan\sqrt{x}\cdot\sec^2\sqrt{x}\cdot\frac{1}{2\sqrt{x}}=$
$\frac{\tan\sqrt{x}\times\sec^2\sqrt{x}}{\sqrt{x}}$

20. B. $f\left(5\right)=\frac{25}{3}$ $f'\left(5\right)=5$ $y-\frac{25}{3}=5\left(x-5\right)\Rightarrow 15x-3y=50$

21. C. John's height on the mountain on day n can be represented by $\sum_{k=1}^{n}\frac{1}{k}$. This is the harmonic series which diverges. How long it will take to reach the top however is another question. If you think about it, the sum of the areas of all the bars in a bar graph using $f\left(x\right)=\frac{1}{x}$ would be the sum of the harmonic series to that point. $\int_1^n f\left(x\right)dx$ over-estimates $\sum_{k=1}^{n}\frac{1}{k}$ and under estimates $\sum_{k=1}^{n-1}\frac{1}{k}$. Therefore for our purposes it will suffice. $\int_1^n\frac{1}{x}dx=100\Rightarrow\ln n=100\Rightarrow n=e^{100}$. In other words, we estimate that it would take approximately e^{100} days to reach the top. This is certainly more than 70 years worth of days (70 * 366 = 25620)!

22. D.

$$\int_0^5 (x-2)^{-2}dx = \lim_{b\to 2^-}\int_0^b (x-2)^{-2} + \lim_{a\to 2^+}\int_a^5 (x-2)^{-2} = \lim_{b\to 2^-}\left[-1\cdot(x-2)^{-1}\right]_0^b + \lim_{a\to 2^+}\left[-1\cdot(x-2)^{-1}\right]_a^5 =$$

$$\lim_{b\to 2^-}\left[\frac{-1}{b-2}-\frac{-1}{0-2}\right] + \lim_{a\to 2^+}\left[\frac{-1}{5-2}-\frac{-1}{a-2}\right] = \text{Does not exist}$$

23. B. $\lim_{x\to-\infty}\frac{3-5x}{\sqrt{14x^2-27x}}\cdot\frac{1/x}{1/x} = \lim_{x\to-\infty}\frac{3-5x}{\sqrt{14x^2-27x}}\cdot\frac{1/x}{-1/\sqrt{x^2}}$ since we're approaching $-\infty$

$$= \lim_{x\to-\infty}\frac{\frac{3}{x}-5}{-\sqrt{14-\frac{27}{x}}} = \frac{-5}{-\sqrt{14}} = \frac{5}{\sqrt{14}}$$

24. C. $\int x^2\sqrt{x+1}dx$ Let $u = x+1 \Rightarrow x = u-1$;

$$\int\left[(u-1)^2\sqrt{u}\right]du = \int\left[u^{\frac{5}{2}}-2u^{\frac{3}{2}}+u^{\frac{1}{2}}\right]du = \frac{2}{7}u^{\frac{7}{2}}-\frac{4}{5}u^{\frac{5}{2}}+\frac{2}{3}u^{\frac{3}{2}}+C =$$

$$\frac{2}{7}(x+1)^{\frac{7}{2}}-\frac{4}{5}(x+1)^{\frac{5}{2}}+\frac{2}{3}(x+1)^{\frac{3}{2}}+C$$

25. C. $\int\frac{1}{e^{-x}+e^x}\cdot\frac{e^x}{e^x}dx = \int\frac{e^x}{1+e^{2x}}dx = \arctan(e^x) + C$

26. A. $\lim_{x\to\infty}\frac{(x+1)(42^{x+1})}{(7x+7)!}\cdot\frac{(7x)!}{x(42^x)} = \lim_{x\to\infty}\frac{(42)}{(7x+1)(7x+2)\cdots(7x+7)} = 0$, therefore the series converges absolutely

27. B. $\lim_{n\to\infty}\left|\frac{(x+5)^{n+1}}{11^{n+1}}\cdot\frac{11^n}{(x+5)^n}\right| < 1 \Rightarrow \left|\frac{x+5}{11}\right| < 1 \Rightarrow -17 < x < 6$ The series does not converge for either endpoint, therefore $x\in(-17,6)$

28. B. $1 = \int_{e-1}^{e^4-1}\frac{k}{x+1}dx = k\left[\ln(x+1)\right]_{e-1}^{e^4-1} = k(4-1) \Rightarrow k = \frac{1}{3}$

29. A. $P(2e-1 \le X \le e^3-1) = \frac{1}{3}\int_{(2e-1)}^{(e^3-1)}\frac{1}{x+1}dx = \frac{1}{3}\left[\ln(x+1)\right]_{(2e-1)}^{(e^3-1)} =$

$$\frac{1}{3}\ln(e^3-1+1)-\frac{1}{3}\ln(2e-1+1) = \frac{1}{3}\cdot 3-\frac{1}{3}(\ln e+\ln 2) = 1-\frac{1}{3}-\frac{1}{3}\ln 2 = \frac{2}{3}-\ln\sqrt[3]{2}$$

30. C. $E(X) =$

$$\frac{1}{3}\int_{e-1}^{e^4-1}\frac{x}{x+1}dx = \frac{1}{3}\left[x-\ln(x+1)\right]_{e-1}^{e^4-1} = \frac{1}{3}\left[\left(e^4-1-\ln(e^4-1+1)\right)-\left(e-1-\ln(e-1+1)\right)\right] =$$

$$\left(\frac{e^4-e-4+1}{3}\right) = \frac{e^4-e}{3} - 1$$