



- 1) E ( $2 - \pi^2 / 4$ )
- 2) B
- 3) D
- 4) D
- 5) E (does not exist)
- 6) D
- 7) B
- 8) A
- 9) D
- 10) B
- 11) C
- 12) D
- 13) B
- 14) C
- 15) A
- 16) D
- 17) E (1)
- 18) E ( $f'(x) = 0$  and concave up)
- 19) B
- 20) C
- 21) B
- 22) A
- 23) B
- 24) E ( $\pi / 2$ )
- 25) A
- 26) C
- 27) B
- 28) A
- 29) A
- 30) D



1) E.  $f'(x) = 2x \sin x + x^2 \cos x \Rightarrow f''(x) = 2 \sin x + 2x \cos x + 2x \cos x - x^2 \sin x \Rightarrow$

$$f''\left(\frac{\pi}{2}\right) = 2 - \frac{\pi^2}{4}.$$

2) B. Using L'Hopital's rule, the expression becomes  $\frac{2x+1}{2x+3} \Rightarrow \frac{3}{5}$ .

3) D. The velocity is given by  $x'(t) = 4t^3 - 2t \Rightarrow x'(2) = 28$ , thus it moves to the right at  $28 \frac{\text{units}}{\text{sec}}$ .

4) D. Plugging 0 into the expression gives

$$f'(x) = 2x \sin x + x^2 \cos x \Rightarrow f''(x) = 2 \sin x + 2x \cos x + 2x \cos x - x^2 \sin x. \text{ L'Hopital's rule is used}$$

twice and the expression becomes  $\frac{20x^3 - 2}{-\cos x} \Rightarrow \frac{-2}{-1} = 2$ .

5) E. Since no limit exists for the function as 1 is approached from the positive side, no limit exists.

6) D.  $\frac{dy}{dx} = \frac{dy}{dt} \left( \frac{dt}{dx} \right) = (2t)(t) = 2t^2. \frac{d^2y}{dx^2} = \frac{d}{dt} \left( \frac{dy}{dx} \right) \left( \frac{dt}{dx} \right) = (4t)(t) = 4t^2 \Rightarrow 4(2)^2 = 16$ .

7) B. The limit is  $-1$  only when the term in the numerator with the highest exponent is 4. This only occurs when  $A = 2$  or  $A = -2$ , thus the sum of all possible values of  $A$  is 0.

8) A.  $f'(x) = 2x \sin(x) \ln(x) + x^2 \cos(x) \ln(x) + x^2 \sin(x) \ln(x) \Rightarrow f'(\pi) = -\pi^2 \ln \pi$

9) D. Between every root of  $f(x)$  there is one point where  $f'(x) = 0$ , thus  $A = n - 1$ . Between every root of  $f'(x)$ , there is one point where  $f''(x) = 0$ , thus  $B = n - 2$ . So  $A + B = 2n - 3$ .

10) B.  $\frac{dy}{dx} = \frac{-\partial x}{\partial y} = \frac{\cos(x) - y}{x + 3y^2}$ .

11) C.  $f(x) = \log_x 9 = \frac{\ln 9}{\ln x} \Rightarrow f'(x) = -\ln 9 \left( \frac{1}{x(\ln x)^2} \right) \Rightarrow f'(e) = \frac{-\ln 9}{e}$ .

12) D.  $g'(x) = \frac{1}{2\sqrt{1+f(x)^2}} (2f(x)f'(x)) = \frac{f(x)f'(x)}{\sqrt{1+f(x)^2}}$ .

13) B.  $f\left(\frac{\sqrt{3}}{2} + 0.1\right) \approx f\left(\frac{\sqrt{3}}{2}\right) + f'\left(\frac{\sqrt{3}}{2}\right)(0.1) = \frac{\pi}{3} + 2(.1) = \frac{\pi}{3} + 0.2$ .

14) C. Using substitution, if  $u = \frac{1}{3x}$ , then the limit becomes  $\lim_{u \rightarrow \infty} \left(1 + \frac{1}{u}\right)^{\frac{3}{4}} = e^{\frac{3}{4}} \Rightarrow A - B = -1$ .

15) A.  $f'(x) = 3x^2 - 18x + 24 \Rightarrow 2(x - 4)(x - 2)$ . Thus the two relative extrema occur at 2 and 4. Since,  $f''(x)$  is negative at 2, this is the relative maximum.

16) D. Plugging in  $h$  directly gives  $\frac{0}{0}$ . Using L'Hopital's rule, the expression becomes  $\frac{2f'(x+2h) + 2f'(x-2h)}{1}$ .

When  $h = 0$  it becomes  $4f'(x)$ .  $f'(x) = 2\pi \cos(2\pi x) \Rightarrow 4f'(0) = 8\pi$ .

17) E.  $y = r \sin \theta = \sin^2 \theta \Rightarrow \frac{dy}{d\theta} = \sin 2\theta \quad x = r \cos \theta = \sin \theta \cos \theta \Rightarrow \frac{dx}{d\theta} = \cos 2\theta$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \left( \frac{d\theta}{dx} \right) = \tan 2\theta \Rightarrow \tan \left( 2 \left[ \frac{\pi}{8} \right] \right) = 1$$



18) E. Using the product rule of derivatives, it can be seen that  $f'(x) = 0$ , thus NOTA is the correct choice.

19) B.  $\frac{dy}{dx} = \frac{y^2}{x^2+1} \Rightarrow \frac{dy}{y^2} = \frac{dx}{x^2+1}$ . Integrating both sides gives  $\frac{-1}{y} = \arctan(x) + C$ . Using the fact that

$$f(1) = -\frac{4}{\pi}, \frac{\pi}{4} = \frac{\pi}{4} + C \Rightarrow C = 0. \text{ Therefore } \frac{-1}{y} = \frac{\pi}{3} \Rightarrow y = \frac{-3}{\pi}.$$

20) C. If the polynomial is written in the form  $Ax^3 + Bx^2 + Cx + D$ , then  $D = 2$ ,  $3A + 2B + C = 13$ ,  $12A + 2B = 18$ , and  $6A = 6$ . So  $A = 1$ ,  $B = 3$ ,  $C = 4$ , and  $D = 2$ . Thus  $f(1) = 10$ .

21) B.  $\frac{d}{dx}[\arcsin(x^2)] = \frac{2x}{x^2\sqrt{x^4-1}} \Rightarrow \frac{2}{x\sqrt{x^4-1}}$ .

22) A. If  $G'(u) = \sin(\sqrt{u}) \Rightarrow \int_x^{x^2} \sin(\sqrt{u}) du = G(x^2) - G(x) \Rightarrow$

$$f'(x) = 2xG'(x^2) - G'(x) = 2x \sin x - \sin \sqrt{x}.$$

23) B.  $f(x) = \frac{3x^3 - 3x^2 - 4x - 3}{x-2} = 3x^2 + 3x + 2 + \frac{1}{x-2}$ . The third derivative of the first three terms is 0.

The third derivative of the last term is the only one that matters. Thus

$$f'''(x) = \frac{-6}{(x-2)^4} \Rightarrow f'''(3) = \frac{-6}{1} = -6.$$

24) E. The expression can be rearranged to  $\lim_{n \rightarrow \infty} \frac{1}{n} \cdot \sum_{i=1}^n \frac{1}{\sqrt{1 - \left(\frac{i}{n}\right)^2}} = \int_0^1 \frac{1}{\sqrt{1-x^2}} = \arcsin(1) - \arcsin(0) = \frac{\pi}{2}$ .

25) B. Using substitution, if  $h = \frac{1}{u}$ , the limit becomes  $\lim_{u \rightarrow \infty} \ln\left(1 + \frac{1}{u}\right)^u = \ln e = 1$ .

26) C. The sum can be rewritten as the integral  $\int_0^1 \sin(\pi x) dx = \frac{-\cos(\pi)}{\pi} + \frac{\cos(0)}{\pi} = \frac{2}{\pi}$ .

27) B. The limit can be rewritten as  $\lim_{n \rightarrow \infty} e^{\frac{1}{n} \sum_{i=0}^n \frac{i}{n}} \Rightarrow e^{\int_0^1 x dx} = e^{\frac{1}{2}}$ .

28) A. Using the first theorem of Pappus, the surface area can be found to equal  $\pi^2 (r_0^2 - r_i^2)$ . Thus

$$\frac{dS}{dt} = \pi^2 (2r_0 \frac{dr_0}{dt} - 2r_i \frac{dr_i}{dt}) \Rightarrow \pi^2 (2(5)(2) - 2(4)(2)) = 4\pi^2.$$

29) A. The water filling the cone forms a cone that has a height twice the length of its radius. Thus

$$h = 2r \Rightarrow V = \frac{\pi h^3}{12} \Rightarrow \frac{dV}{dt} = \frac{\pi h^2}{4} \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = 2 \cdot \frac{4}{9\pi} = \frac{8}{9\pi}.$$

30) D. The limit can be rewritten as  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \sqrt{1 - \left(\frac{i}{n}\right)^2} = \int_0^1 \sqrt{1-x^2} dx$ . This integral is equal to a quarter of

the area of a unit circle, thus the answer is  $\frac{\pi}{4}$ .