



Mu Alpha Theta
2006 National Convention

Limits and Derivatives
Mu Division

1) If $f(x) = x^2 \sin x$, determine $f''\left(\frac{\pi}{2}\right)$.

- A) 0 B) 1 C) π D) 2π E) NOTA

2) Evaluate $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 + 3x - 4}$.

- A) $-\frac{3}{5}$ B) $\frac{3}{5}$ C) $-\frac{1}{2}$ D) $\frac{1}{2}$ E) NOTA

3) A particle moves along the x-axis and its position is given by $x(t) = t^4 - t^2 - 1$ where t represents time in seconds. Which of the following best describes its velocity at $t = 2$.

- A) $11 \frac{\text{units}}{\text{sec}}$ to the left. B) $11 \frac{\text{units}}{\text{sec}}$ to the right.
C) $28 \frac{\text{units}}{\text{sec}}$ to the left. D) $28 \frac{\text{units}}{\text{sec}}$ to the right. E) NOTA

4) Evaluate $\lim_{x \rightarrow 0} \frac{x^5 - x^2}{\cos x - 1}$.

- A) -2 B) -1 C) 1 D) 2 E) NOTA

5) Evaluate $\lim_{x \rightarrow 1} \sqrt{1 - x^2}$.

- A) 0 B) $\frac{1}{2}$ C) $-\frac{1}{2}$ D) $\sqrt{2}$ E) NOTA

6) If $y = t^2 + 1$ and $x = \ln t$, where $t > 0$, determine $\frac{d^2y}{dx^2}$ when $t = 2$.

- A) $\frac{1}{8}$ B) $\frac{1}{16}$ C) 8 D) 16 E) NOTA

7) If $\lim_{x \rightarrow \infty} \frac{x^A + x^{2-A} + x^{2A}}{4 - x^4} = -1$, where A is an integer, find the sum of all possible values of A .

- A) -2 B) 0 C) 2 D) 4 E) NOTA

8) If $f(x) = x^2 \sin(x) \ln(x)$, where $x > 0$, determine $f'(\pi)$.

- A) $-\pi^2 \ln \pi$ B) $\pi^2 \ln \pi$ C) 0 D) -2 E) NOTA



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9) Given $f(x) = (x-1)(x-2)(x-3)\dots(x-n)$, where n is an integer greater than or equal to four, let A be equal to the number of values of x for which $f'(x) = 0$ and let B represent the number of values for which $f''(x) = 0$. Find $A + B$ in terms of n .

- A) $n-1$ B) $2n-1$ C) $2n-2$ D) $2n-3$ E) NOTA

10) If $xy + y^3 - \sin x = 0$, find $\frac{dy}{dx}$.

- A) $\frac{y - \cos x}{x + 3y^2}$ B) $\frac{\cos x - y}{x + 3y^2}$ C) $y + 3y^2 - \cos x$ D) $\frac{x + 3y^2}{y - \cos x}$ E) NOTA

11) Given that $f(x) = \log_x 9$, find $f'(e)$.

- A) $\frac{1}{e \ln 9}$ B) $\frac{-1}{e \ln 9}$ C) $\frac{-\ln 9}{e}$ D) $\ln 9$ E) NOTA

12) If $g(x) = \sqrt{1 + f(x)^2}$ find $g'(x)$ in terms of $f(x)$.

- A) $\frac{f(x)}{\sqrt{1 + f(x)^2}}$ B) $-\frac{f(x)}{\sqrt{1 + f(x)^2}}$ C) $\frac{f'(x)}{\sqrt{1 + f(x)^2}}$ D) $\frac{f(x)f'(x)}{\sqrt{1 + f(x)^2}}$ E) NOTA

13) Given that $f'(x) = \frac{1}{\sqrt{1-x^2}}$ and $f\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$, find the linear approximation of $f\left(\frac{\sqrt{3}}{2} + 0.1\right)$.

- A) $\frac{\pi}{3} + 0.1$ B) $\frac{\pi}{3} + 0.2$ C) $\frac{\pi}{3} + 0.75$ D) $\frac{\pi}{3} - 0.1$ E) NOTA

14) The limit $\lim_{x \rightarrow 0^+} (3x+1)^{\frac{1}{4x}}$ can be written in the form $e^{\frac{A}{B}}$ where A and B are relatively prime positive integers. Find $A - B$.

- A) -7 B) 7 C) -1 D) 1 E) NOTA

15) For the function $f(x) = x^3 - 9x^2 + 24x + 10$, there exists one relative maximum. For what value of x does the relative maximum occur?

- A) 2 B) -2 C) -4 D) 4 E) NOTA

16) If $f(x) = \sin(2\pi x)$, determine $\lim_{h \rightarrow 0} \frac{f(x+2h) - f(x-2h)}{h}$, when $x = 0$.

- A) π B) 2π C) 4π D) 8π E) NOTA



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17) Given the polar graph $r = \sin \theta$, determine the slope of the line tangent to the graph at the point where

$$\theta = \frac{\pi}{8} \text{ and } r \neq 0.$$

- A) 3 B) $\frac{\sqrt{3}}{3}$ C) $\sqrt{3}$ D) 6 E) NOTA

18) If $f(x) = (x+1)^2(x-1)(x-3)^2$, which of the following statements correctly describes the graph of $f(x)$ at the point $x = 3$?

- A) $f(x)$ is increasing and concave up. B) $f(x)$ is decreasing and concave up.
C) $f(x)$ is increasing and concave down. D) $f(x)$ is decreasing and concave down E) NOTA

19) If $f'(x) = \frac{[f(x)]^2}{x^2 + 1}$, and $f(1) = -\frac{4}{\pi}$, find $f(\sqrt{3})$.

- A) $-\frac{1}{\pi}$ B) $-\frac{3}{\pi}$ C) $-\frac{4}{\pi}$ D) $-\frac{6}{\pi}$ E) NOTA

20) For a third degree polynomial $f(x)$, $f(0) = 2$, $f'(1) = 13$, $f''(2) = 18$, and $f'''(7) = 6$. Find $f(1)$.

- A) 2 B) 6 C) 10 D) 12 E) NOTA

21) Find $\frac{d}{dx} [\arcsin(x^2)]$.

- A) $\frac{2}{x^2\sqrt{x^2-1}}$ B) $\frac{2}{x\sqrt{x^4-1}}$ C) $\frac{1}{x^2\sqrt{x^4-1}}$ D) $\frac{2}{x\sqrt{x^2-1}}$ E) NOTA

22) If $f(x) = \int_x^{x^2} \sin(\sqrt{u})du$ for $x \geq 0$, find $f'(x)$.

- A) $2x \sin x - \sin \sqrt{x}$ B) $\sin x - \sin \sqrt{x}$
C) $\cos x - \cos \sqrt{x}$ D) $2x \cos x - \cos \sqrt{x}$ E) NOTA

23) If $f(x) = \frac{3x^3 - 3x^2 - 4x - 3}{x - 2}$, find $f'''(3)$.

- A) 6 B) -6 C) 7 D) -7 E) NOTA



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24) Evaluate $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{\sqrt{n^2 - i^2}}$.

- A) 0 B) $\frac{e}{2}$ C) π D) e E) NOTA

25) Evaluate, $\lim_{h \rightarrow \infty} \ln\left(\sqrt[h]{1 + \frac{h}{x}}\right)$, when $x = 1$.

- A) 0 B) 1 C) e D) $\frac{1}{e}$ E) NOTA

26) Evaluate $\lim_{n \rightarrow \infty} \left[\frac{1}{n} (\sin(\frac{\pi}{n}) + \sin(\frac{2\pi}{n}) + \sin(\frac{3\pi}{n}) + \dots + \sin(\frac{n\pi}{n})) \right]$.

- A) 2 B) $\frac{1}{\pi}$ C) $\frac{2}{\pi}$ D) $\frac{4}{\pi}$ E) NOTA

27) Evaluate. $\lim_{n \rightarrow \infty} \prod_{i=1}^n \sqrt[n]{e^{\frac{i}{n}}}$

- A) 1 B) \sqrt{e} C) e D) e^2 E) NOTA

28) A torus has an outer radius r_o , and an inner radius r_i . If r_o and r_i are increasing at a rate of $\frac{2cm}{sec}$ each,

find the rate of change of the surface area of the torus in $\frac{cm^2}{sec}$ when $r_o = 5cm$ and $r_i = 4cm$.

- A) $4\pi^2 \frac{cm^2}{sec}$ B) $9\pi^2 \frac{cm^2}{sec}$ C) $20\pi^2 \frac{cm^2}{sec}$ D) $80\pi^2 \frac{cm^2}{sec}$ E) NOTA

29) An inverted cone with a height of $10cm$ and a radius of $5cm$ is being filled with water at a rate of $2\frac{cm^3}{sec}$.

Find the rate of change of the depth of the water, when the depth is $3cm$.

- A) $\frac{8}{9\pi} \frac{cm}{sec}$ B) $\frac{4}{9\pi} \frac{cm}{sec}$ C) $\frac{2}{5\pi} \frac{cm}{sec}$ D) $\frac{4}{15\pi} \frac{cm}{sec}$ E) NOTA

30) Evaluate $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\sqrt{n^2 - i^2}}{n^2}$.

- A) 1 B) $\frac{\pi}{8}$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{4}$ E) NOTA