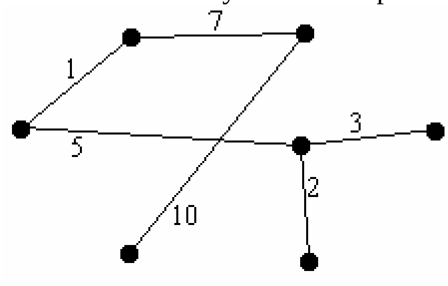
1. C
2. D
3. B
4. C
5. B
6. E
7. D
8. A
9. D
10. B
11. B
12. C
13. B
14. A
15. D
16. B
17. D
18. A
19. C
20. D
21. B
22. C
23. A
24. B
25. A
26. E
27. A
28. E
29. A
30. C

1. $((p \wedge q) \rightarrow \sim r) \vee \sim s \equiv \sim (p \wedge q) \vee \sim r \vee \sim s \equiv \sim p \vee \sim q \vee \sim r \vee \sim s \equiv \sim (p \wedge q \wedge r \wedge s)$ ANS. C

2. Here is the only minimum spanning tree for this graph. Its total weight is 28. ANS. D



3. The explicit representation for $A(0,n)$ is given as $n+1$. We can then proceed as follows:
$A(1,0) = A(0,1) = 2$
$A(1,n) = A(0, A(1,n-1)) = A(1,n-1)+1$
$\therefore A(1,n) = n+2$
$A(2,0) = A(1,1) = 3$
$A(2,n) = A(1, A(2,n-1)) = A(2,n-1)+2$
$\therefore A(2,n) = 2n+3$
$A(3,0) = A(2,1) = 5$
$A(3,n) = A(2, A(3,n-1)) = 2A(3,n-1)+3$
$\therefore A(3,n) = 2^{n+3}-3$
ANS. B

4. Cathy and Eve are knights, and the rest are knaves. Any other assignment leads to a contradiction. Hence, there are 2 knights. ANS. C

5. Argument B is a textbook example of the converse error, a common logical fallacy. Hence, it is not a valid argument. The other arguments are valid, despite the fact that some of their premises may be false. ANS. B

6. The number of zeros is determined by the number of fives in the prime factorization of 2006! (This is because the number of twos in the prime factorization of a factorial will always be greater than the number of fives.) Hence, the answer is given by
$\left\lfloor \frac{2006}{5} \right\rfloor + \left\lfloor \frac{2006}{25} \right\rfloor + \left\lfloor \frac{2006}{125} \right\rfloor + \left\lfloor \frac{2006}{625} \right\rfloor = 401+80+16+3 = 500$. ANS. E.

7. There are a total of 64 different graphs with 4 nodes, all of which are equally likely since p=0.5. (There are 6 different possible edges, each of which can either exist or not exist, 2^6 possibilities.) None of the graphs with 0, 1, or 2 edges can be connected. All of the graphs with 4, 5, or 6 edges must be connected. Of the 20 graphs with 3 edges (${}_6C_3$), there are only 4 which are not connected. Hence, the total number of connected graphs is $({}_6C_3 - 4) + {}_6C_4 + {}_6C_5 + {}_6C_6 = 16+15+6+1 = 38$, and the probability that the Bernoulli graph is connected is $\frac{38}{64} = \frac{19}{32}$. ANS D.

8. There are 13 ways to choose the denomination for the 3 matching cards and then 12 ways to choose the denomination for the 2 matching cards. Hence, there are 13*12=156 ways to choose the denominations. Within the triple, there are ${}_4C_3 = 4$ ways to choose the suits. Within the pair, there are ${}_4C_2 = 6$ ways to choose the suits. Hence, there are 13*12*4*6 = 3744 different hands which are full houses. ANS. A.

9. Since 37 is prime, the positive integers less than 37 form a commutative group under modulo 37 multiplication. Hence, every positive integer in the set {1, 2, 3, …, 36} has a multiplicative inverse from the same set under modulo 37 multiplication. Thus, the answer is 36. ANS. D

10. This recursion can easily be solved as $r_n = 3n + 1$. Thus $r_{25} = 3(25) + 1 = 76$. ANS. B.

11. Choice B cannot be equivalent. Given *x*, which is an element of set C but not set A, we have $x \notin A \cap (B \cup C)$ but $x \in (A \cap B) \cup C$. The others are equivalent by application of the distributive property and/or DeMorgan's Laws. ANS. B

12. $(A \cup B) \cap (A \cup B^C) \cap A^C = A \cap A^C$. This is the empty set. ANS. C.

13. Proof by induction and contradiction are common. Similarly, since the contrapositive of a statement is logically equivalent to the statement, proof of the contrapositive would also be a valid method of proof. Proof by contraindication, though, is nonsense. "Contraindication: A factor that renders the administration of a drug or the carrying out of a medical procedure inadvisable." ANS. B

14. The greatest common divisor of two positive integers will be a natural number (even if it is 1). ANS. A.

15. The values of the function must be g(a)=1, g(b)=3, g(c)=g(d)=2, g(e)=g(f)=5. ANS. D.

16. Since the domain of h is restricted to the natural numbers, this function is an injection. (That is, it is a one-to-one function.) It is not a surjection, though, because it is not onto – e.g. there is no *x* in the domain of natural numbers for which h(x) = 3. A function must be both an injection and a surjection in order to be a bijection. ANS. B

17. Recall that an equivalence relation is a binary relation that is reflexive, symmetric, and transitive, and recall that a binary relation on A is formally defined as a subset of A × A. R^C and $(R \cup R')^C$ cannot be equivalence relations because they are not reflexive. $R \cup R'$ is reflexive and symmetric, but it may not be transitive. Only $R \cap R'$ is reflexive, symmetric, and transitive. ANS. D.

18. If we calculate the quantity $2^{n+2} + 3^{2n+1}$ for n=0, then we obtain 7. This immediately tells us that the gcd must be either 1 or 7. To check if $2^{n+2} + 3^{2n+1}$ is divisible by 7 for all n, we proceed by induction, noting that $2^{(n+1)+2} + 3^{2(n+1)+1} = 2^{n+3} + 3^{2n+3} = 2 \cdot 2^{n+2} + 9 \cdot 3^{2n+1} = 2(2^{n+2} + 3^{2n+1}) + 7 \cdot 3^{2n+1}$. Thus, obviously, if $2^{n+2} + 3^{2n+1}$ is divisible by 7, then so is $2^{(n+1)+2} + 3^{2(n+1)+1}$. Hence, the greatest common divisor of this sequence is 7. ANS. A.

19. Once you recognize (through truth tables or other means) that $(a \to b) \to a$ is logically equivalent to *a* and that $(a \to b) \to b$ is logically equivalent to $a \vee b$, it becomes clear that $((a \to b) \to a) \to a$ must be the only tautology here. ANS. C

20. The elements in P(A × B) are sets of ordered pairs. The elements in P(A) × P(B) are ordered pairs of sets. Hence, neither set is a subset of the other. ANS. D

21. The negation of $\forall x, \exists y$ s.t. $x \rightarrow y$ is $\exists x$ s.t. $\forall y, x \wedge \sim y$. ANS. B

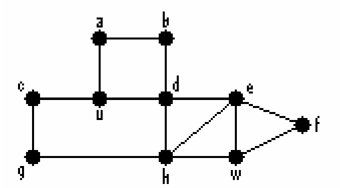
22. A minimal sequence of steps is: (1) fill C, (2) transfer C to A, (3) fill C, (4) transfer C to B, (5) fill C, (6) transfer C to B, (7) transfer C to A. ANS. C

23. B is the definition of completeness and C is the definition of transitivity, hence they must apply. It is easy to derive D from completeness and transitivity. A, however, is false, because not all complete and transitive binary relations are asymmetric. ANS. A

24. The GCD is 17. Hence, the LCM = 1989*2006/17 = 234,702. Therefore, the sum is 234,719. ANS. B

25. Since 2 + 2 ≠ 5, the statement is vacuously true. ANS. A

26. A Euler path traverses each edge exactly once. Hence, any Euler path for this graph must contain 14 edges. With respect to the diagram below, one such path is u,c,g,h,d,b,a,u,d,e,f,w,e,h,w. ANS. E



27. Referencing the drawing in the previous solution, one possible three-coloring of the graph would make the following assignment of colors: red-{a,c,f,h}, green – {b,u,e,g}, blue – {d,w}. Because of the configuration of nodes {e,f,w}, a two-coloring of the graph is impossible. ANS. A

28. Translating the postfix notation to standard (infix) notation produces (-2 / 4) + ((3 + 5) - 2) = 5.5. ANS. E

29. Matrix multiplication is not commutative. If A and B are matricies, then AB and BA need not even have the same dimensions (or one or both may not exist). Matrix multiplication is associative and distributive, and it has the identity property (with an n x n identity element usually called "the identity matrix"). ANS. A

30. The innermost multiplication is carried out n(n+1)(n+2)/6 times. Hence, the algorithm is of time complexity order O(n^3). ANS. C