There are some variations in discrete math symbols. For this test:
- $p \wedge q$ means "p and q,"
- $p \vee q$ means "p or q,"
- $\sim p$ means "not p,"
- $p \rightarrow q$ means "if p then q,"
- $f : A \rightarrow B$ means that f is a function with domain A and co-domain B, it should always be clear from the context which of these two meanings of "$\rightarrow$" we intend,
- $p \leftrightarrow q$ means "p if and only if q," and
- A^C means "the complement of a A" where A is a set.

Other notation used is either standard (i.e. used in the vast majority of discrete math textbooks) or will be defined as needed.

1. Which of the following is logically equivalent to $((p \wedge q) \rightarrow \sim r) \vee \sim s$?

    A. $(p \wedge q) \rightarrow (r \wedge s)$

    B. $(p \wedge q \wedge \sim r) \vee \sim s$

    C. $\sim (p \wedge q \wedge r \wedge s)$

    D. $(p \wedge q) \vee \sim r \vee \sim s$

    E. NOTA

2. Consider the weighted graph shown. What is the sum of the weights of the edges which constitute a minimum spanning tree?
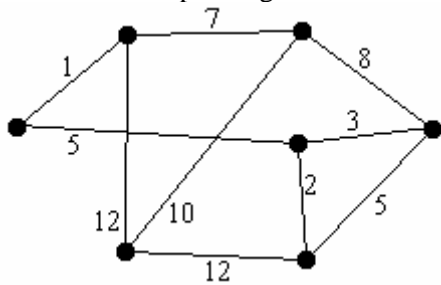
    A. 8   B. 65   C. 30   D. 28   E. NOTA

3. The Ackerman Function, $A(m,n)$, is defined for all non-negative integers m and n as follows.

$$A(0,n) = n+1 \qquad\qquad n \geq 0$$
$$A(m,0) = A(m-1,1) \qquad\qquad m > 0$$
$$A(m,n) = A(m-1, A(m,n-1)) \quad n > 0, m > 0$$

    Which of the following is an explicit representation for $A(3,n)$?

    A. $n+1$ B. $2^{n+3} - 3$ C. $\dfrac{n^2 - n + 4}{2}$ D. $n! + 1$ E. NOTA

4. You are on an island in which there are only two types of people: knights, who always tell the truth and knaves, who never tell the truth. You meet with a group of 6 people, Alice, Bob, Cathy, David, Eve, and Fred. They tell you the following:

    Alice says, "None of us is a knight." Bob says, "At least 3 of us are knights." Cathy says, "At most 3 of us are knights." David says, "Exactly 5 of us are knights." Eve says, "Exactly 2 of us are knights." Fred says, "Exactly 1 of us is a knight."

    How many knights are there in this group of 6?

    A. 6   B. 4   C. 2   D. It is impossible to determine.   E. NOTA

5. Which of the following is not a valid argument?
   A. All Mu Alpha Theta members must take calculus. You are a Mu Alpha Theta member. Therefore, you must take calculus.
   B. If a computer program is correct, then it does not contain syntax errors. The Mathcad program does not contain syntax errors. Therefore, the Mathcad program is correct.
   C. If the product of two numbers is zero, then at least one of the numbers is zero. For particular numbers x and y, neither x nor y is equal to zero. Therefore, the product of x and y is not zero.
   D. If Harry is a Gryffindor then he is brave. If Harry is brave, then he will defeat Lord Voldemort. Therefore, if Harry is a Gryffindor then he will defeat Lord Voldemort.
   E. NOTA

6. If written out as an integer in decimal notation, 2006! ends with how many zeros?
   A. 201   B. 401   C. 497   D. 501   E. NOTA

7. Let B(n,p) be a Bernoulli random graph with n nodes and edge probability p. This is a graph which has n vertices, but the number and position of the edges is determined at random. Specifically, between any two distinct vertices, there is probability p that the edge exists, independent of the existence or nonexistence of any other edge. What is the probability that B(4,0.5) is a connected graph?
   A. 5/16   B. 1/64   C. 1/2   D. 19/32   E. NOTA

8. A poker hand is created by drawing 5 cards at random from a standard 52 card deck. How many distinct poker hands are full houses? A full house is a hand containing 3 cards of one denomination and 2 cards of a second denomination. The order of the five cards does not matter; in other words, two hands which contain the same cards in different orders are not distinct.
   A. 3744   B. 44928   C. 156   D. 936   E. NOTA

9. How many of the positive integers less than 37 have an integer multiplicative inverse, also less than 37, under modulo 37 multiplication?
   A. 6   B. 37   C. 3   D. 36   E. NOTA

10. Suppose that $r_k = 2r_{k-1} - r_{k-2}$ for all integers $k \geq 2$, that $r_0 = 1$, and that $r_1 = 4$. What is the value of r_{25}?
    A. 99   B. 76   C. 75   D. 256   E. NOTA

11. Which of the following sets is not equal to $A \cap (B \cup C)$?
    A. $(A \cap B) \cup (A \cap C)$ B. $(A \cap B) \cup C$ C. $A \cap (B^C \cap C^C)^C$
                     D. $(A^C \cup B^C)^C \cup (A^C \cup C^C)^C$ E. NOTA

12. Which of the following sets must be equal to the empty set?
    A. $A \cup A^C$ B. $A \cap B^C$ C. $(A \cup B) \cap (A \cup B^C) \cap A^C$
                     D. $(A \cup B) \cap (A \cup B^C) \cap (A^C \cup B)$ E. NOTA

13. Which of the following is not a valid method of proof?
    A. Proof of the Contrapositive
    B. Proof by Contraindication
    C. Proof by Induction
    D. Proof by Contradiction
    E. NOTA

14. Consider gcd(x,y) as a function with domain $Z^+ \times Z^+$ which returns the greatest common divisor of x and y (where Z^+ is the set of positive integers). What is the range of gcd(x,y)?
   A. Z^+ B. The set of prime numbers   C. Z (the set of integers)   D. $Z^+ \cup \{0\}$ E. NOTA

15. Suppose there is a function g: {a,b,c,d,e,f} $\rightarrow$ {1,2,3,4,5,6} and that the inverse image of the function for certain subsets $B \subseteq \{1,2,3,4,5,6\}$ is given by the following table.

B	$g^{-1}(B)$
{1,2,3}	{a,b,c,d}
{1,3}	{a,b}
{1,2,5}	{a,c,d,e,f}
{1,3,6}	{a,b}

What is the value of g(f)?
   A. 6   B. 2   C. 1   D. 5   E. NOTA

16. Suppose there is a function $h: N \rightarrow N$, where N is the set of natural numbers, which is given by the rule $h(x) = x^2$. Which of the following statements is true?
   A. h is a bijection.   B. h is an injection.   C. h is a surjection.
          D. h is neither an injection nor a surjection   E. NOTA

17. Suppose that R and R' are equivalence relations on a set A. Which of the following must also be an equivalence relation on A?
   A. R^C B. $R \cup R'$ C. $(R \cup R')^C$ D. $R \cap R'$ E. NOTA

18. What is is the greatest common factor of $2^{n+2} + 3^{2n+1}$ for all nonnegative integers n?
   A. 7   B. 37   C. 1   D. 5   E. NOTA

19. Which of the following is a tautology?
   A. $(a \rightarrow b) \rightarrow a$ B. $(a \rightarrow b) \rightarrow b$ C. $((a \rightarrow b) \rightarrow a) \rightarrow a$
          D. $((a \rightarrow b) \rightarrow a) \rightarrow b$ E. NOTA

20. Suppose that A and B are sets, that P(A) denotes the power set of A (that is, the set of all subsets of A), and that A × B denotes the Cartesian product of A and B. Which of the following statements is true?
   A. P(A) × P(B) = P(A × B)   B. P(A) × P(B) ⊂ P(A × B)   C. P(A × B) ⊂ P(A) × P(B)
          D. P(A × B) ⊄ P(A) × P(B) and P(A) × P(B) ⊄ P(A × B)   E. NOTA

21. Which of the following is the negation of $\forall x, \exists y$ s.t. $x \rightarrow y$? (Note: s.t. denotes "such that".)
   A. $\forall x, \exists y$ s.t. $y \rightarrow x$ B. $\exists x$ s.t. $\forall y, x \wedge \sim y$ C. $\forall x, \exists y$ s.t. $x \wedge \sim y$
   D. $\exists y$ s.t. $\forall x, x \rightarrow y$ E. NOTA

22. You are given empty containers A, B, and C which hold exactly 8, 5, and 3 gallons, respectively, and an unlimited supply of water. A "step" consists of one of the following operations: (1) **fill** a chosen container with water, (2) **transfer** water from one container to another (until the first is empty or the second is full, whichever comes first), (3) **empty** a chosen container (onto the ground). What is the minimum number of steps required to end with exactly 4 gallons of water in container A?
   A. 9   B. 6   C. 7   D. 8   E. NOTA

23. A binary relation on a set is said to be a *weak preference relation* if it is complete and transitive. If $\triangleright$ is a weak preference relation on X, then which of the following statements is not necessarily true? Assume that x, y, z ∈ X.

    A. $(x \triangleright y) \rightarrow (y \not\triangleright x)$ B. $(x \triangleright y) \vee (y \triangleright x)$

    C. $((x \triangleright y) \wedge (y \triangleright z)) \rightarrow (x \triangleright z)$ D. $(x \triangleright z) \rightarrow ((x \triangleright y) \vee (y \triangleright z))$ E. NOTA
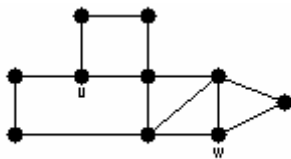
24. What is the sum of the greatest common divisor and the least common multiple of 2006 and 1989?

    A. 3,989,935   B. 234,719   C. 12,155   D. 3,995   E. NOTA

25. Evaluate the following proposition: "If 2+2 = 5 then 4+7 = 11."

    A. True   B. False   C. Cannot be determined   D. Null   E. NOTA

26. Consider the graph shown below. How many edges are there in the shortest Euler path from vertex u to vertex w?

    A. 3   B. 4   C. 6   D. There is no Euler path from u to w.   E. NOTA

27. What is the chromatic number of the graph in the previous problem?

    A. 3   B. 4   C. 7   D. 10   E. NOTA

28. Evaluate the following postfix notation expression: -2 4 / 3 5 + 2 - +

    A. -10.5   B. -5.5   C. 4   D. 9.5   E. NOTA

29. Which of the following properties is not satisfied by matrix multiplication?

    A. commutative property
    B. associative property
    C. distributive property
    D. identity property
    E. NOTA

30. An algorithm is given below. What is the time complexity of this algorithm?

```
for i = 1 to n
   for j = 1 to i
      for k = 1 to j
         x = i * j * k
      next k
   next j
next i
```

    A. $O(n^2)$ B. O(3n)   C. $O(n^3)$ D. O(n log n)   E. NOTA