1. C
2. C
3. A
4. E
5. C
6. B
7. B
8. C
9. B
10. D
11. B
12. E
13. D
14. A
15. C
16. D
17. Thrown Out
18. B
19. B
20. Thrown Out
21. Thrown Out
22. E
23. A
24. C
25. D
26. C
27. B
28. D
29. E
30. A

1. $f'(x)=\dfrac{(2x+2)(x+7)-1\left(x^2+2x-3\right)}{(x+7)^2}=\dfrac{x^2+14x+17}{(x+7)^2}$.

$f''(x)=\dfrac{(2x+14)(x+7)^2-2(x+7)\left(x^2+14x+17\right)}{(x+7)^4}$. $f''(-3)=1$. **C**

2. $\ln y=\dfrac{\ln x}{\ln 2}\cdot\ln 3$. $\dfrac{y'}{y}=\dfrac{\ln 3}{\ln 2}\cdot\dfrac{1}{x}$. $y'(2)=\dfrac{\ln 3}{\ln 2}\cdot\dfrac{3}{2}=\log_4 27$. **C**

3. Using integration by parts, $\displaystyle\int_1^{\sqrt{e}} x\ln x\,dx=\left[x^2\ln x-x^2\right]_1^{\sqrt{e}}-\int_1^{\sqrt{e}} x\ln x\,dx+\int_1^{\sqrt{e}} xdx$.

$\displaystyle 2\int_1^{\sqrt{e}} x\ln x\,dx=\frac{e}{2}-e+1+\frac{e}{2}=1$. $\displaystyle\int_1^{\sqrt{e}} x\ln x\,dx=\frac{1}{2}$. **A**

4. $f(x)=2\sin(x)$. $f'(x)=2\cos x=0$. $x=\dfrac{\pi k}{2},k\in N$ $A=2$, $B=-2$. $A-B=4$. **E**

5. $\displaystyle\sum_{i=0}^{2006}\binom{2006}{i}\cdot(-2)^i=\sum_{i=0}^{2006}\binom{2006}{i}\cdot(-2)^i(1)^{2006-i}=(-2+1)^{2006}=1$ by the binomial theorem. **C**

6. $f(x)=y=\sqrt{x+y}$. $f(2)=2$. $y'=\dfrac{1+y'}{2\sqrt{x+y}}$. $y'(2)=\dfrac{1+y'(2)}{4}$. $y'(2)=\dfrac{1}{3}$. **B**

7. $f(x)=\dfrac{1}{\left[x^2\right]}=\begin{cases}1 & 1\le x<\sqrt{2}\\ \dfrac{1}{2} & \sqrt{2}\le x<\sqrt{3}\\ \dfrac{1}{3} & \sqrt{3}\le x<2\end{cases}$. $\displaystyle\int_1^2\frac{1}{\left[x^2\right]}dx=\sqrt{2}-1+\frac{\sqrt{3}-\sqrt{2}}{2}+\frac{2-\sqrt{3}}{3}=\frac{\sqrt{3}}{6}+\frac{\sqrt{2}}{2}-\frac{1}{3}$. **B**

8. $f(-x)=f(x)$ so $-f'(-x)=f'(x)$ and hence $f'(-x)=-f'(x)$. Therefore $f'(x)$ is odd. Similarly $g'(x)$ is even. $h'(-5)=\dfrac{f'(-5)\left[g(-5)\right]^2-2g(-5)g'(-5)f(-5)}{\left[g(-5)\right]^4}=\dfrac{-2\cdot(-5)^2-2\cdot 5\cdot -1\cdot 6}{[-5]^4}=\dfrac{2}{125}$. **C**

9. Let $u=\sqrt{x}$. Then $u^2=x$ and $2u\,du=dx$. $\displaystyle\int_1^4\frac{e^{\sqrt{x}}}{\sqrt{x}}dx=\int_1^2\frac{e^u}{u}2u\,du=\left[2e^u\right]_1^2=2e(e-1)$. **B**

10. $R(x)=100x$. $MR(x)=100$. $MC(x)=.1x-10$. $MR=MC$ implies $x=1100$. Profits are given by $R(1100)-C(1100)=100\cdot 1100-\left[.05\cdot 1100^2-10\cdot 1100-1000\right]=$ $100\cdot 1100-\left[45\cdot 1100\right]+1000>-1000$. Therefore 1100 cases of widgets will be produced. **D**

11. $P\left(x+y>xy\right)=P\left(y>\frac{x}{x-1}\right)=\frac{1}{5\cdot 10}\int_5^{10}\frac{x}{x-1}\,dx=\frac{1}{50}\int_5^{10}\left(1+\frac{1}{x-1}\right)dx=\frac{1}{50}\left[x+\ln\left(x-1\right)\right]_5^{10}=$ $\frac{1}{10}+\frac{1}{25}\ln\frac{3}{2}$. **B**

12. $\lim\limits_{x\to 0}\frac{axe^x}{\sin(bx)}=\lim\limits_{x\to 0}\frac{axe^x+ae^x}{b\cos(bx)}=\frac{a}{b}=1$. $a-b=0$. **E**

13. $0<\frac{1}{x^4+2x^2+1}<\ln\left(x^2+1\right)<x\cdot\sqrt[3]{x^4+1}<x\cdot 2^x$ for all $x\in[2,5]$. **D**

14. $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\frac{i}{i^2+n^2}\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\frac{\frac{i}{n}\cdot\frac{1}{n}}{\frac{i^2}{n^2}+\frac{n^2}{n^2}}=\int_0^1\frac{x}{x^2+1}\,dx=\frac{1}{2}\left[\ln\left(x^2+1\right)\right]_0^1=\frac{1}{2}\ln 2=\ln\sqrt{2}$. **A**

15. $n=\sum\limits_{i=1}^{\infty}\left\lfloor\frac{2006}{2^i}\right\rfloor=1003+501+256+125+62+31+15+7+3+1=1998$. **C**

16. $\int_{-\pi}^{\pi}\left|\sin(x)\right|dx=2\int_0^{\pi}\sin(x)\,dx=2\left[-\cos(x)\right]_0^{\pi}=4$. **D**

17. $\int_{-\infty}^{\infty}\frac{1}{\sqrt{\pi}}e^{-\frac{x^2}{3}}dx=\sqrt{\frac{3}{2}}\cdot\sqrt{2}\int_{-\infty}^{\infty}\frac{1}{\sqrt{\frac{3}{2}}\sqrt{2\pi}}e^{-\frac{1}{2}\cdot\frac{x^2}{\left(\sqrt{\frac{3}{2}}\right)^2}}dx=\sqrt{\frac{3}{2}}\cdot\sqrt{2}\cdot 1=\sqrt{3}$. **B**

18. Using L'Hopital's Rule with the Second Fundamental Theorem of Calculus,
$$\lim_{x\to 0}\left(\frac{1}{x\cos(x)}\int_{x^2}^{x}e^{t^2}\,dt\right)=\lim_{x\to 0}\frac{e^{x^2}-2xe^{x^2}}{\cos(x)-x\sin(x)}=1.$$ **B**

19. Using differentials, $2.01^n\approx 2^n+.01\cdot n2^{n-1}$. $2.01^n-2^n\approx .01\cdot n\cdot 2^{n-1}>10$. $n\cdot 2^n>1000$. The smallest value of n is $n=8$. **B**

20. Brian's initial vertical velocity is $64\sin(30°)=32$. $a=-32$, $v=-32t+32$, $s=-16t^2+32t$. Alex's initial vertical velocity is $v\sin(45°)=\frac{v\sqrt{2}}{2}$. $a=-32$, $v=-32t+\frac{v\sqrt{2}}{2}$, $s=-16t^2+\frac{v\sqrt{2}}{2}t$. If the balls collide, then $-16t^2+32t=-16t^2+\frac{v\sqrt{2}}{2}t$ for some t and $v=32\sqrt{2}$. **C**

21. If the balls land at the same time, their vertical velocities must be the same as in the last problem, and hence $v=32\sqrt{2}$. Brian's horizontal velocity is $64\cos\left(30°\right)=32\sqrt{3}$. Alex's horizontal velocity is $32\sqrt{2}\cos\left(45°\right)=32$. Since the balls are in the air the same amount of time, it follows that Brian's will travel further. **B**

22. $A=2\pi\int_0^1 (x+3)\left(\sqrt{x}-x^2\right)dx = 2\pi\left[\frac{2}{5}x^{\frac{5}{2}}-\frac{x^4}{4}+2x^{\frac{3}{2}}-x^3\right]_0^1 = \frac{23\pi}{10}$. **C**

23. Notice $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} \cos\left(-\frac{\pi}{6}\right) & \sin\left(-\frac{\pi}{6}\right) \\ -\sin\left(-\frac{\pi}{6}\right) & \cos\left(-\frac{\pi}{6}\right) \end{bmatrix}$ is a rotation matrix that would rotate a curve $30°$ counterclockwise. Since $2006 \bmod 12 = 2$, $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}^{2006} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}^{2} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$. **A**

24. $y^2\,dy = x\,dx$. $\frac{y^3}{3}=\frac{x^2}{2}+C$. $9=8+C$. $C=1$. $\frac{\left[y(0)\right]^3}{3}=1$. $y(0)=\sqrt[3]{3}$. **C**

25. $\int_0^\infty Cxe^{-x^2}\,dx=1$. $\lim_{M\to\infty} -\frac{C}{2}\left[e^{-x^2}\right]_0^M = \frac{C}{2}=1$. $C=2$. **D**

26. Since $2x+4y=300$, $U=75x^2-\frac{x^3}{2}$. $150x-\frac{3}{2}x^2=0$. $x=100$. **C**

27. $\binom{9}{5}+\binom{9}{6}+\binom{9}{7}+\binom{9}{8}+\binom{9}{9}=126+84+36+9+1=256$. **B**

28. $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$. $2x\ln x + x = \frac{d}{d\left[\frac{1}{x}\right]}x^2\ln x\cdot -\frac{1}{x^2}$. $\frac{d}{d\left[\frac{1}{x}\right]}x^2\ln x = -x^3\left(2\ln x+1\right)$. **D**

29. $\lim_{x\to\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2+2x}\right)\frac{\left(\sqrt{x^2+3x}+\sqrt{x^2+2x}\right)}{\left(\sqrt{x^2+3x}+\sqrt{x^2+2x}\right)} = \lim_{x\to\infty}\frac{x}{\sqrt{x^2+3x}+\sqrt{x^2+2x}} =$

$\lim_{x\to\infty}\frac{x}{\sqrt{x^2+3x}+\sqrt{x^2+2x}}\cdot\frac{\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)} = \lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{3}{x}}+\sqrt{1+\frac{2}{x}}} = \frac{1}{2}$. **E**

30. $\int_{-a}^{a}\left(2\sqrt{a^2-x^2}\right)^2 dx = \int_{-a}^{a}\left(4a^2-4x^2\right)dx = \left[4a^2x-\frac{4}{3}x^3\right]_{-a}^{a} = \frac{16a^3}{3}=144$. $a=3$.