1. B
2. C
3. E
4. C
5. A
6. B
7. A
8. A
9. C
10. B
11. A
12. D
13. C
14. D
15. E
16. D
17. E
18. C
19. D
20. B
21. A
22. B
23. A
24. D
25. A
26. B
27. A
28. C
29. B
30. A

1. From Pappus's Theorem, the Volume V = 2πAr, where A is the area of the shape and r is the average radius of rotation or the distance from the axis of rotation to the centroid. So for A = 4 and r = 4, the V = 32π **B**

2. The formula for the volume of the largest cone works out to be $V_{cone} = 8/27 V_{sphere}$ so V = 81π (8/27) = 24π **C**

3. A would be 1 and 5, B -4,5 C => 2,7, D is made up. **E**

4. By definition dodecagon **C**

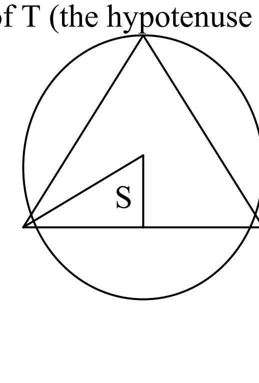
5. let h be the height of the cone, R be the radius of the cone and r be the radius of the sphere. By similar triangles, $\frac{h-r}{r} = \frac{\sqrt{h^2+R^2}}{R}$ which can be re written $R^2 = \frac{hr^2}{(h-2r)}$. The volume of the cone is now $\frac{\pi R^2 h}{3} = \frac{\pi h^2 r^2}{3(h-2r)}$ After taking derivative and setting equal to zero $\frac{h^2r^2 - 4hr^3}{(h-2r)^2} = 0$, thus h=4r.

therefore the volume is $\left.\frac{8\pi r^3}{3}\right|_{r=5} = \frac{1000\pi}{3}$ **A**

6. $$\int_0^1 (1-x^2)\,dx \Big/ \int_0^1 x^2\,dx = 2$$ **B**

7. $$2\pi\int_0^1 x(1-x^2)\,dx \Big/ 2\pi\int_0^1 x(x^2)\,dx = 1$$ **A**

8. The maximum area of the base of this tetrahedron T must be an equilateral triangle (max triangle inscribed in a circle), and the base of T will be perpendicular to its height (r + x) where r is the radius of the tetrahedron and x is the distance from the base of T to the center of the sphere. Thus the volume of T $V(x) = \frac{1}{3}(Area_{base})(r+x)$ So we must find the area of the base of T. Let's look at the right triangle formed by leg x and hypotenuse r, with other leg on the base of T (the hypotenuse of triangle S in the diagram. That length = $\sqrt{r^2 - x^2}$ and so the area of the base can be found to be $\frac{3\sqrt{3}(r^2-x^2)}{4}$ plugging t back into our equation for V(x) and maximizing the function yields x = r/3 and so $\frac{8r^3\sqrt{3}}{27}$ is to be the maximum volume of a tetrahedron in sphere with radius r. So our volume is $512\sqrt{3}$ **A**



9. $\left[\pi\left(\frac{r\sqrt{2}}{2}\right)^2 2\left(\frac{r\sqrt{2}}{2}\right)-\frac{4}{3}\pi\left(\frac{r\sqrt{2}}{2}\right)^3\right] \Big/ \frac{4}{3}\pi r^3 = \frac{1}{4\sqrt{2}} = \frac{\sqrt{2}}{8}$ **C**

10. $\left[2\pi\left(\frac{r\sqrt{2}}{2}\right)r\sqrt{2}+4\pi\left(\frac{r\sqrt{2}}{2}\right)^2\right] \Big/ 4\pi r^2 = 1$ **B**

11. $\int_0^8 \sqrt{1+\left(\frac{2}{3}x^{\frac{-1}{3}}\right)^2}\,dx+12$ can be solved if changed to an integral with respect to y.

$y = x^{\frac{2}{3}} \Rightarrow x = y^{\frac{3}{2}}$, the new y-limits will be $y=0$ to $y=4$, so we can

evaluate $\int_0^4 \sqrt{1+\left(\frac{3}{2}y^{\frac{1}{2}}\right)^2}\,dy+12 = \int_0^4 \sqrt{1+\frac{9}{4}y}\,dy+12$, let $u = 1+\frac{9}{4}y$, so

$\frac{4}{9}\int_1^{10} \sqrt{u}\,du+12 = \frac{80\sqrt{10}+316}{27}$ **A**

12. $\frac{d}{dx}\int_0^x \sin(t)dt \Big|_{x=\frac{\pi}{2}} = 1$ **D**

13. $\frac{\pi}{3}\left(\frac{3}{4}h\right)^2 h = V, \frac{9\pi}{16}h^2\frac{dh}{dt} = \frac{dV}{dt} = 1, h = 4, \frac{dh}{dt} = \frac{1}{9\pi}$ **C**

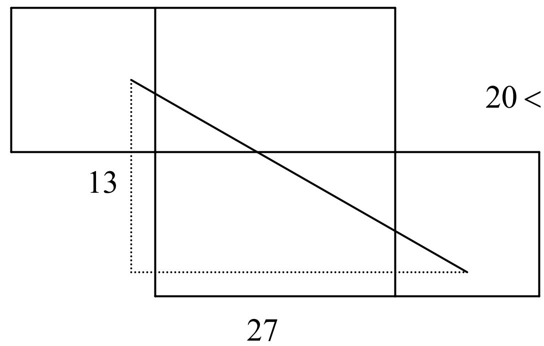
14. $\left(\frac{3}{4}h\right)^2 \frac{\pi}{3}h = 1, h = 4\sqrt[3]{12}$ **D**

15. We are only given information about the ant's rate moving vertically and horizontally. The shortest distance between the ant and his intended destination is $\sqrt{4^2+\pi^2}$, but we have no information about the ant's rate of movement diagonally, so we need to find it using related rates. We obtain $d^2 = h^2+\pi^2r^2$, where *d*=shortest distance traversed by ant, *h* = height of cylinder, *r* =radius of cylinder. Taking the derivatives with respect to time in order to find $\frac{dd}{dt}$,

$2d\frac{dd}{dt} = 2h\frac{dh}{dt}+2\pi^2r\frac{dr}{dt}$, and substituting, we get, $\frac{dd}{dt} = \frac{4\cdot 2+\pi^2\cdot 1\cdot 1}{\sqrt{16+\pi^2}} = \frac{8+\pi^2}{\sqrt{16+\pi^2}}$. The time it takes the ant to travel should be $\frac{\text{distance}}{\text{rate}} = \frac{\sqrt{4^2+\pi^2}}{\frac{8+\pi^2}{\sqrt{16+\pi^2}}} = \frac{16+\pi^2}{8+\pi^2}$. So, the answer is $\frac{16+\pi^2}{8+\pi^2}$ **E**

16. s = r-x $\lim_{x\to r}\frac{\pi r^2-\pi x^2-\pi(r-x)^2}{\pi(r-x)^2} = \lim_{x\to r}\frac{(r-x)(r+x)-(r-x)^2}{(r-x)^2} = \lim_{x\to r}\frac{r+x-r+x}{r-x} \to \infty$ **D**

17.



$20 < \sqrt{13^2 + 27^2} < 30$ **E**

18. $s\sqrt{3} = 12, V = \left(\frac{12}{\sqrt{3}}\right)^3 = 192\sqrt{3}$ **C**

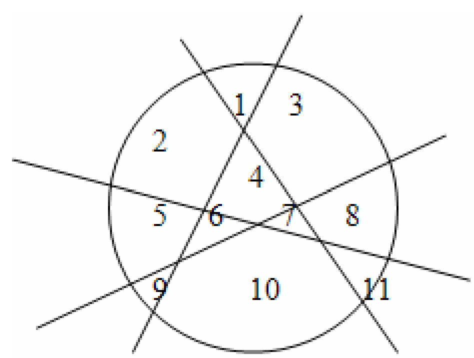
19. Solve $\frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = 3\pi r^2$ **D**

20. Maximize $\frac{1}{2} 4 \cdot 4 \cdot \sin\theta \to 8$ **B**

21. V + F – 2 = E, Letting V = 7 and E = 12, (7) + F – 2 = (12) so F = 7**A**

22. If we were to cut the Mobius strip widthwise, a rectangular ribbon of width *y* would be produced. The total surface area of this object could be found by measuring the length of the ribbon, multiplying it by *y*, and then doubling the result to account for the two sides of the ribbon. So we are multiplying *y* by twice the length of the ribbon. But since *x* forms a complete cycle around the edge of the Mobius strip, it actually traverses the length of the rectangular ribbon twice. Thus the total surface area is *xy*. **B**

23. 11 **A (**see diagram to the right)



24. Triangle ABC is similar to Triangle FEC $\frac{EF}{AB} = \frac{CF}{AC} \to EF = \frac{3}{4} \cdot 5 = 3.75$ **D**

25. $\frac{6}{3}\left(81 + 16 + \sqrt{81 \cdot 16}\right) = 266$ **A**

26. Notice that r is an altitude of triangle JMK, whose base has measure s. Therefore the area of triangle JMK is ½ rs, and the area of JKLM, which is twice as large as JMK, is rs. Now notice that the length of the long diagonal of JKLM is 2R, while the short diagonal is defined as d. Therefore the area of JKLM is ½ (2R)(d) = Rd. Hence, 72 = $rRds$ = (rs)(Rd) = (area of JKLM)^2. So the area of JKLM is $6\sqrt{2}$. **B**

27. $k = \frac{1}{radius} = \frac{1}{4}$ **A**

28. $Radius_{inscribedcircle} = \frac{2Area_{triangle}}{Perimeter_{triangle}} = 1, Area_{circle} = \pi$ **C**

29. **B** by definition of incenter

30. The curve in question is a semicircle of radius 2 centered at the origin. Since the semicircle is defined over the interval [-2, 2], we are computing the area of half of this semicircle, which is π. **A**