1. D
2. B
3. A
4. E
5. E
6. C
7. B
8. D
9. E
10. B or E
11. A or C
12. C
13. D
14. A
15. D
16. D
17. D
18. B
19. D
20. A
21. D
22. E
23. B
24. E
25. B
26. E
27. C
28. B
29. B
30. D

1. From Pappus's Theorem, the Volume V = 2πAr, where A is the area of the shape and r is the average radius of rotation or the distance from the axis of rotation to the centroid. So for A = 4 and r = 4, the V = 32π **B**

2. The formula for the volume of the largest cone works out to be $V_{cone} = 8/27V_{sphere}$ so

V = 81π (8/27) = 24π **C**

3. A would be 1 and 5, B -4,5 C => 2,7, D is made up. **E**

4. By definition dodecagon **C**

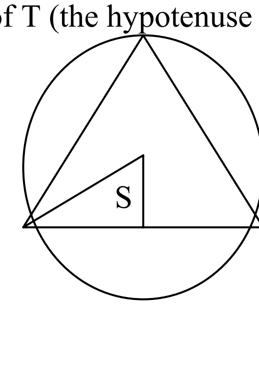
5. let h be the height of the cone, R be the radius of the cone and r be the radius of the sphere. By similar triangles, $\frac{h-r}{r} = \frac{\sqrt{h^2+R^2}}{R}$ which can be re written $R^2 = \frac{hr^2}{(h-2r)}$. The volume of the cone is now $\frac{\pi R^2 h}{3} = \frac{\pi h^2 r^2}{3(h-2r)}$ After taking derivative and setting equal to zero $\frac{h^2r^2 - 4hr^3}{(h-2r)^2} = 0$, thus h=4r.

therefore the volume is $\left.\frac{8\pi r^3}{3}\right|_{r=5} = \frac{1000\pi}{3}$ **A**

6. $\int_0^1 (1-x^2)\,dx \Big/ \int_0^1 x^2\,dx = 2$ **B**

7. $2\pi\int_0^1 x(1-x^2)\,dx \Big/ 2\pi\int_0^1 x(x^2)\,dx = 1$ **A**

8. The maximum area of the base of this tetrahedron T must be an equilateral triangle (max triangle inscribed in a circle), and the base of T will be perpendicular to its height (r + x) where r is the radius of the tetrahedron and x is the distance from the base of T to the center of the sphere. Thus the volume of T $V(x) = \frac{1}{3}(Area_{base})(r+x)$ So we must find the area of the base of T. Let's look at the right triangle formed by leg x and hypotenuse r, with other leg on the base of T (the hypotenuse of triangle S in the diagram. That length = $\sqrt{r^2 - x^2}$ and so the area of the base can be found to be $\frac{3\sqrt{3}\left(r^2 - x^2\right)}{4}$ plugging t back into our equation for V(x) and maximizing the function yields x = r/3 and so $\frac{8r^3\sqrt{3}}{27}$ is to be the maximum volume of a tetrahedron in sphere with radius r. So our volume is $512\sqrt{3}$ **A**



9. $\left[\pi\left(\frac{r\sqrt{2}}{2}\right)^2 2\left(\frac{r\sqrt{2}}{2}\right)-\frac{4}{3}\pi\left(\frac{r\sqrt{2}}{2}\right)^3\right] \Big/ \frac{4}{3}\pi r^3 = \frac{1}{4\sqrt{2}} = \frac{\sqrt{2}}{8}$ **C**

10. $\left[2\pi\left(\frac{r\sqrt{2}}{2}\right)r\sqrt{2}+4\pi\left(\frac{r\sqrt{2}}{2}\right)^2\right] \Big/ 4\pi r^2 = 1$ **B**

11. $\int_0^8 \sqrt{1+\left(\frac{2}{3}x^{\frac{-1}{3}}\right)^2}\,dx+12$ can be solved if changed to an integral with respect to y.

$y = x^{\frac{2}{3}} \Rightarrow x = y^{\frac{3}{2}}$, the new y-limits will be $y=0$ to $y=4$, so we can

evaluate $\int_0^4 \sqrt{1+\left(\frac{3}{2}y^{\frac{1}{2}}\right)^2}\,dy+12 = \int_0^4 \sqrt{1+\frac{9}{4}y}\,dy+12$, let $u=1+\frac{9}{4}y$, so

$\frac{4}{9}\int_1^{10}\sqrt{u}\,du+12 = \frac{80\sqrt{10}+316}{27}$ **A**

12. $\frac{d}{dx}\int_0^x \sin(t)dt\Big|_{x=\frac{\pi}{2}} = 1$ **D**

13. $\frac{\pi}{3}\left(\frac{3}{4}h\right)^2 h = V, \frac{9\pi}{16}h^2\frac{dh}{dt} = \frac{dV}{dt} = 1, h = 4, \frac{dh}{dt} = \frac{1}{9\pi}$ **C**

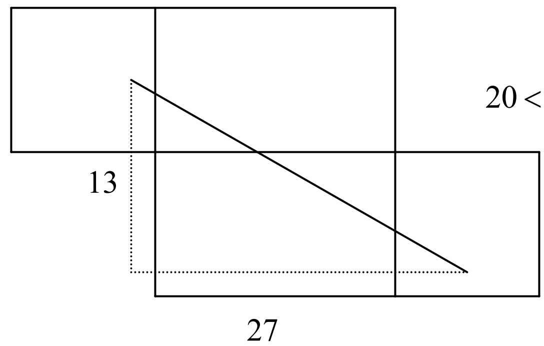
14. $\left(\frac{3}{4}h\right)^2\frac{\pi}{3}h = 1, h = 4\sqrt[3]{12}$ **D**

15. We are only given information about the ant's rate moving vertically and horizontally. The shortest distance between the ant and his intended destination is $\sqrt{4^2+\pi^2}$, but we have no information about the ant's rate of movement diagonally, so we need to find it using related rates. We obtain $d^2 = h^2+\pi^2 r^2$, where d=shortest distance traversed by ant, h = height of cylinder, r =radius of cylinder. Taking the derivatives with respect to time in order to find $\frac{dd}{dt}$,

$2d\frac{dd}{dt} = 2h\frac{dh}{dt}+2\pi^2 r\frac{dr}{dt}$, and substituting, we get, $\frac{dd}{dt} = \frac{4\cdot 2+\pi^2\cdot 1\cdot 1}{\sqrt{16+\pi^2}} = \frac{8+\pi^2}{\sqrt{16+\pi^2}}$. The time it takes the ant to travel should be $\frac{\text{distance}}{\text{rate}} = \frac{\sqrt{4^2+\pi^2}}{\frac{8+\pi^2}{\sqrt{16+\pi^2}}} = \frac{16+\pi^2}{8+\pi^2}$. So, the answer is $\frac{16+\pi^2}{8+\pi^2}$ **E**

16. s = r-x $\lim_{x\to r}\frac{\pi r^2-\pi x^2-\pi(r-x)^2}{\pi(r-x)^2} = \lim_{x\to r}\frac{(r-x)(r+x)-(r-x)^2}{(r-x)^2} = \lim_{x\to r}\frac{r+x-r+x}{r-x} \to \infty$ **D**

17.



$20 < \sqrt{13^2 + 27^2} < 30$ **E**

18. $s\sqrt{3} = 12, V = \left(\frac{12}{\sqrt{3}}\right)^3 = 192\sqrt{3}$ **C**

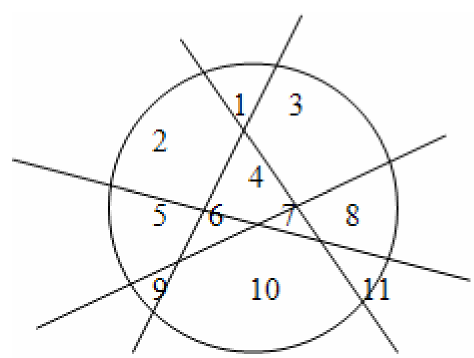
19. Solve $\frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = 3\pi r^2$ **D**

20. Maximize $\frac{1}{2}4 \cdot 4 \cdot \sin\theta \rightarrow 8$ **B**

21. V + F – 2 = E, Letting V = 7 and E = 12, (7) + F – 2 = (12) so F = 7**A**

22. If we were to cut the Mobius strip widthwise, a rectangular ribbon of width *y* would be produced. The total surface area of this object could be found by measuring the length of the ribbon, multiplying it by *y*, and then doubling the result to account for the two sides of the ribbon. So we are multiplying *y* by twice the length of the ribbon. But since *x* forms a complete cycle around the edge of the Mobius strip, it actually traverses the length of the rectangular ribbon twice. Thus the total surface area is *xy*. **B**

23. 11 **A (**see diagram to the right)



24. Triangle ABC is similar to Triangle FEC $\frac{EF}{AB} = \frac{CF}{AC} \rightarrow EF = \frac{3}{4} \cdot 5 = 3.75$ **D**

25. $\frac{6}{3}\left(81 + 16 + \sqrt{81 \cdot 16}\right) = 266$ **A**

26. Notice that r is an altitude of triangle JMK, whose base has measure s. Therefore the area of triangle JMK is ½ rs, and the area of JKLM, which is twice as large as JMK, is rs. Now notice that the length of the long diagonal of JKLM is $2R$, while the short diagonal is defined as d. Therefore the area of JKLM is ½ $(2R)(d) = Rd$. Hence, $72 = rRds = (rs)(Rd) =$ (area of JKLM)^2. So the area of JKLM is $6\sqrt{2}$. **B**

27. $k = \frac{1}{radius} = \frac{1}{4}$ **A**

28. $Radius_{inscribedcircle} = \frac{2Area_{triangle}}{Perimeter_{triangle}} = 1, Area_{circle} = \pi$ **C**

29. **B** by definition of incenter

30. The curve in question is a semicircle of radius 2 centered at the origin. Since the semicircle is defined over the interval [-2, 2], we are computing the area of half of this semicircle, which is π . **A**