**Mu Alpha Theta**
**2006 National Convention**

**Answers**
**What Would DaVinci Do?**
**Open Division**

1. Leonardo
2. Denon
3. “Try to figure out the problem”
4. The Last Supper
5. Mona Lisa
6. 415
7. Heron’s Formula
8. 4
9. 41
10. $\frac{16r^3}{3}$
11. 2/3
12. 9
13. 1700
14. 2.5
15. 18
16. 1
17. 9/35
18. 4
19. 10
20. 10/11
21. 1/5
22. 2
23. 1/6
24. 24
25. 49

1. Leonardo
2. Denon
3. It says, "Try to figure out the problem."
4. The Last Supper
5. Mona Lisa
6. 415 (florins)
7. Heron's Formula
8. 4—In the end, the amount of wine in one jug must equal the amount of water in the other jug. So, both jugs must be ½ wine and ½ water. Using the first jug, after the 1st transfer it has (x -2 wine): (x total). The 2nd transfer removes wine in that ratio and adds wine in the ratio 2:x from the other jug. Thus the amt of wind at the end is $(x-2)-\left(\frac{x-2}{x}\right)\cdot 2+\left(\frac{2}{x}\right)\cdot 2,$ which also must equal $\frac{x}{2}$ …so, x = 4.
9. 41—If you roll the cylindrical surface and wire onto a plane, you can see that the 9", the circumference (10 x 4"=40") and the wire (L) form a right triangle, with L being the hypotenuse. Hence, L = 41, by the Pythagorean Theorem.
10. $\frac{16r^3}{3}$ - In the volume common to both cylinders place a sphere of radius r having its centre at the intersection of the two axes of the cylinders. Now take any plane section through the cylinders. This will give you a square cross section of the area common to both cylinders which circumscribes a circle as the cross section of the sphere. Now, clearly the volume of the sphere is the sum of all circular cross sections and the volume of the solid common to both cylinders is the sum of all square cross sections. The ratio of the volume of the sphere to the volume of the solid is therefore the same as the ratio of the area of a circle to the area of a circumscribed square. So $\frac{\frac{4\pi r^3}{3}}{x}=\frac{\pi}{4}$ and $x=\frac{16r^3}{3}$.
11. 2/3—$\left(\frac{1\cdot 2\cdot 4+2\cdot 4\cdot 8+3\cdot 6\cdot 12+\ldots}{1\cdot 3\cdot 9+2\cdot 6\cdot 18+3\cdot 9\cdot 27+\ldots}\right)^{1/3}=\left(\frac{1\cdot 2\cdot 4(1^3+2^3+3^3+\ldots)}{1\cdot 3\cdot 9(1^3+2^3+3^3+\ldots)}\right)^{1/3}=\left(\frac{8}{27}\right)^{1/3}=2/3$
12. 9—If (n)(n + 2)(n + 4)(n + 6) = $m^2$, then $(n^2+6n+4)^2=m^2+16.$ But only 9 and 0 are squares of the form $a^2-16$, and since $m^2$ is odd, then the square sought must be 9.
13. 1700—Boat A left one shore, traveled 700 ft and met boat B. Together, they traveled the width of the river. Boat A continues across to the opposite shore then goes back 400 feet, where it met boat B again. Together they had traveled 3 times the width of the river. Since speeds were constant, boat A traveled 3 times 700 feet (2100 ft). The width was 400 ft less than the distance A traveled, that is, 1700 feet.
14. 2.5—Two round trips made the first way would take 3 hours, thus covering the distance between home and his shop twice walking and twice riding. Therefore, he could make the round trip by walking in (3 – ½) hours = 2 ½ hours.
15. 18—The common result must have 7 & 11 as factors, then the number is 7 + 11 or 18. The general solution of ( x – k) k = (x – m)m is k + m.

16. 1—The number 76, formed by the last 2 digits is divisible by 4. The difference between 73 (the sum of the even placed digits) and 17 +45 (the sum of the odd placed digits) is divisible by 11, regardless of the order in which the blanks are filled. The sum of all the digits 90 + 45, is divisible by 9. It follows that the number is divisible by (4)(11)(9) or 396, so the probability is 1.

17. 9/35—Let F = fair day, R = rainy day, S = snowy day, T= commuter on time, and L = commuter late. Given that P(F) = 0.6, P( R) = 0.3, P(S) = 0.1, P(T|F) = 0.8, P(T|R) = 0.6 and P(T|S) = 0.4. We need to find P(R|T), which = $\frac{P(R \cap T)}{P(R)}$. Remember,
$P(R \cap T) = P(T \cap R) = P(T \mid R) \cdot P(R) = (0.6)(0.3) = .18$ For P(T), we use
$P(T) = P(T \cap F) + P(T \cap R) + P(T \cap S) = P(T \mid F) \cdot P(F) + P(T \mid R) \cdot P(R) + P(T \mid S) \cdot P(S)$
$= (0.8)(0.6) + (0.6)(0.3) + (0.4)(0.1) = 0.70 \Rightarrow P(R \mid T) = \frac{0.18}{0.70} = \frac{9}{35}$
18. 4—Let the fly be x meters from the ceiling. Then the fly and point P form a major diagonal of the rectangular prism with dimensions, 1, 8, and x. Therefore, $1^2 + 8^2 + x^2 = 9^2$ and x = 4.
19. 10 secs—Let the center of the wheel be O and the rider travels from point A to point B (an arc on the wheel). Let the midpoint of OA be C, and since OA = 20, then OC = CA = 10. OB also = 20 (radius). This forms right triangle OCB, with OC equal to ½ the hypotenuse OB. This makes angle COB = 60 degrees. Since the wheel turns thru an angle of 360 degrees in 60 seconds, the time required to turn thru an angle of 60 degrees is 10 seconds.
20. $\frac{10}{11}$ --The optimal way of distributing the balls is: 1W in box 1, 1W in box 2 & the rest (16W & 6B) in box 3. The probability of drawing a W ball is $\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)\left(\frac{16}{22}\right) = \frac{10}{11}$
21. $\frac{1}{5}$ -- We need to select one face of one 1 by 1 cube at random, and each one is equally likely.
There are 6(25) faces that have been painted red (the surface area of the original cube) and 6(125) faces in total (125 small cubes, each with 6 faces). Therefore, $P(\mathit{rolling\ a\ red\ face}) = \frac{6(25)}{6(125)} = \frac{1}{5}$
22. 2-- Since no three vertices of the cube are connected to each other, two colors may be used.
23. $\frac{1}{6}$ -- The edge of the octahedron is the hypotenuse of an isosceles right triangle with leg measuring ½, so the edge length is $\sqrt{2}$/2. The octahedron is made up of two square pyramids with height ½ joined at their bases. Since the volume of a square pyramid is V = ⅓$a^2$h, where a is the side of the square and h is the height of the pyramid, $2\left(\frac{1}{3}\right)\left(\frac{\sqrt{2}}{2}\right)^2\left(\frac{1}{2}\right) = \frac{1}{6}$
24. 24—A block with exactly 4 faces in contact with neighboring blocks will have 2 faces exposed. On each of the 12 edges of the original cube there are 2 such blocks (in the center of each edge). Thus the total number is 12 x 2 = 24.
25. 49—Imagining the cube in a 3-D plane, we look at those parallel to the x-axis—9 lines. Considering the y-axis and the z-axis gives 18 more lines. There are 2 diagonals each in the 3 planes parallel to the xy plane (6 total). Considering the yz plane and the xz plane gives 12 more. The cube itself has 4 diagonals. So, 27 + 18 + 4 = 49.