Question # 1

Alpha School Bowl

MAΘ 2006 National Convention

Bert's Backyard Ballers host a game of touch football everyday after school. For simplicity, they adopt the following scoring system: A touchdown is worth 7 points. A safety is worth 3 points.

Find the sum of all positive integer scores that are impossible in Bert's system.

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Question # 2

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$\sin 20° + \sin 40° = \sin \theta$

Find the degree measure of θ. $(0° \le \theta \le 90°)$

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Question # 3

Alpha School Bowl

MAΘ 2006 National Convention

Let the intersection of the following pairs of lines define the vertices of quadrilateral **ABCD**.

A : $\begin{aligned} 3x+4y&=-2 \\ x+3y&=1 \end{aligned}$ **B :** $\begin{aligned} 3x-4y&=2 \\ x-y&=1 \end{aligned}$

C : $\begin{aligned} 6x-6y&=-6 \\ 10x-11y&=-5 \end{aligned}$ **D :** $\begin{aligned} -4x-6y&=6 \\ 5x+7y&=-5 \end{aligned}$

Find the area of **ABCD**.

Question # 3

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Question # 4

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Evaluate $\sum_{k=1}^{2006} k - \sqrt{\sum_{k=1}^{2006} k^3}$.

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Evaluate $\sum_{k=1}^{2006} k - \sqrt{\sum_{k=1}^{2006} k^3}$

Question # 5

Alpha School Bowl

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Given: $\tan\phi = 2/3$, ϕ lies in quadrant I

$\frac{\sin(2\phi)}{1+\cos(2\phi)} = \frac{a}{b}$, where a and b are relatively prime.

Find $a + b$.

Question # 5

Alpha School Bowl

MAΘ 2006 National Convention

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$\frac{\sin(2\phi)}{1+\cos(2\phi)} = \frac{a}{b}$, where a and b are relatively prime.

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Question # 6

Alpha School Bowl

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A : Number of intersections between the plots for $x^2 + y^2 + 10x + 21 = 0$ & $y^2 - x - 3 = 0$.

B : Distance between the foci of the conic defined by $9x^2 - 16y^2 = 144$.

C : Length of the latus rectum of the conic defined by $y^2 = 16x$.

D : Area bounded the conic defined by $4x^2 + 9y^2 + 8x - 54y + 49 = 0$.

Find $A + B + C + D/\pi$.

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Find $A + B + C + D/\pi$.

Question # 7

Alpha School Bowl

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Let $i = \sqrt{-1}$

A : $\sum_{n=1}^{2006} i^n$

B : $|8 - 15i|$

C : sum of the complex roots: $x^2 + 4x + 20 = 0$

D : $[\cos(\pi/12) + i\sin(\pi/12)]^{-6}$

Find $A + B + C + D$.

Question # 7

Alpha School Bowl

MAΘ 2006 National Convention

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C : sum of the complex roots: $x^2 + 4x + 20 = 0$

D : $[\cos(\pi/12) + i\sin(\pi/12)]^{-6}$

Find $A + B + C + D$.

Question # 8

Alpha School Bowl

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A : $\lim_{x\to\infty}\dfrac{3x+4}{x^2-2006x}$

B : $\lim_{x\to 3}\dfrac{x^3-3x^2-x+3}{2x^2-11x+15}$

C : $\lim_{x\to 0}\dfrac{2x-2}{1-x}$

D : $\lim_{x\to-\infty}\left[\dfrac{10^{10^{100}}}{x}+\dfrac{8x^2-4x+2}{2x^2-4x+8}\right]$

Find $A + B + C + D$.

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Alpha School Bowl

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D : $\lim_{x\to-\infty}\left[\dfrac{10^{10^{100}}}{x}+\dfrac{8x^2-4x+2}{2x^2-4x+8}\right]$

Find $A + B + C + D$.

Question # 9

Alpha School Bowl

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Solve each inequality over $0 \le \theta \le 2\pi$.

A : $\sin\theta > \cos\theta$ ***B*** : $\sec\theta > \csc\theta$

Find the sum of the lengths of all subintervals included in the solution set for $\boldsymbol{A} \cap \boldsymbol{B}$.

Question # 9

Alpha School Bowl

MAΘ 2006 National Convention

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Find the sum of the lengths of all subintervals included in the solution set for $\boldsymbol{A} \cap \boldsymbol{B}$.

Question # 10

Alpha School Bowl

MAΘ 2006 National Convention

A : Constant term in the expansion of $\left(2x^4 + x^{-2}\right)^6$

B : product of the roots of $x^3 - 7x + 6 = 0$

C : permutations of the letters in the word COLORADO

D : handshakes exchanged between 8 people if everyone shakes hands with each other once.

Find BC / AD .

Question # 10

Alpha School Bowl

MAΘ 2006 National Convention

A : Constant term in the expansion of $\left(2x^4 + x^{-2}\right)^6$

B : product of the roots of $x^3 - 7x + 6 = 0$

C : permutations of the letters in the word COLORADO

D : handshakes exchanged between 8 people if everyone shakes hands with each other once.

Find BC / AD .

Question # 11

Alpha School Bowl

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The nautical mile is the length of an arc of one minute on a great circle of the earth. It is the standard unit for measuring distances in marine and air navigation. Approximating one "land" or "standard" mile as being one nautical mile (and the earth as a perfect sphere), the earth's radius is $\mathbf{A/\pi}$.

Find A.

Question # 11

Alpha School Bowl

MAΘ 2006 National Convention

The nautical mile is the length of an arc of one minute on a great circle of the earth. It is the standard unit for measuring distances in marine and air navigation. Approximating one "land" or "standard" mile as being one nautical mile (and the earth as a perfect sphere), the earth's radius is $\mathbf{A/\pi}$.

Find A.

Question # 12

Alpha School Bowl

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An orange die and a blue die are cast. Each is fair and six-sided, with the numbers 1 thru 6.

A : The probability of rolling at least one 6.

B : The probability that a sum of 11 is rolled, given that a 6 is rolled on the orange die

C : Probability that the number rolled on the orange die is less than the number on the blue die.

D : Probability that the product of the numbers rolled is greater than 1 and less than 5.

Find $A + B + C + D$.

Question # 12

Alpha School Bowl

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A : The probability of rolling at least one 6.

B : The probability that a sum of 11 is rolled, given that a 6 is rolled on the orange die

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D : Probability that the product of the numbers rolled is greater than 1 and less than 5.

Find $A + B + C + D$.

Question # 13

Alpha School Bowl

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$$\frac{3x^2+5x+1}{(x-1)(x+2)^2} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

Find $\frac{9AB}{C}$.

Question # 13

Alpha School Bowl

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$$\frac{3x^2+5x+1}{(x-1)(x+2)^2} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

Find $\frac{9AB}{C}$.

Question # 14

Alpha School Bowl

MAΘ 2006 National Convention

ΔLHS is an equilateral triangle whose sides have a length of 4.

A : area of ΔLHS

B : apothem of ΔLHS

C : radius of the circumscribed circle

D : radius of the inscribed circle

Find ABC / D

Question # 14

Alpha School Bowl

MAΘ 2006 National Convention

ΔLHS is an equilateral triangle whose sides have a length of 4.

A : area of ΔLHS

B : apothem of ΔLHS

C : radius of the circumscribed circle

D : radius of the inscribed circle

Find ABC / D

Question # 15

Alpha School Bowl

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Let ***A*** be the smaller angle formed by the intersection of $x - y = 0$ & $\sqrt{3}x - y + 2006 = 0$.

Let ***B*** be the angle between the vectors $\langle 4,3,2 \rangle$ & $\langle 1,0,-2 \rangle$.

$\sin(A+B) = \dfrac{\sqrt{a}+\sqrt{b}}{c}$, where neither *a* nor *b* are not divisible by the square of any prime.

Find a + b + c.

Question # 15

Alpha School Bowl

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Let ***A*** be the smaller angle formed by the intersection of $x - y = 0$ & $\sqrt{3}x - y + 2006 = 0$.

Let ***B*** be the angle between the vectors $\langle 4,3,2 \rangle$ & $\langle 1,0,-2 \rangle$.

$\sin(A+B) = \dfrac{\sqrt{a}+\sqrt{b}}{c}$, where neither *a* nor *b* are not divisible by the square of any prime.

Find a + b + c.