



Mu Alpha Theta
2006 National Convention

Alpha Ciphering
Answers

P. $\cot 2\theta$

1. 6

2. 51

3. $\pm\sqrt{2}$

4. 63π

5. 6

6. 7

7. $\frac{4}{5}$

8. $\frac{18}{47}$

9. $\frac{22}{5}$ or $4\frac{2}{5}$ or 4.4

10. $\sqrt{15}$

11. $2k^2$

12. 125

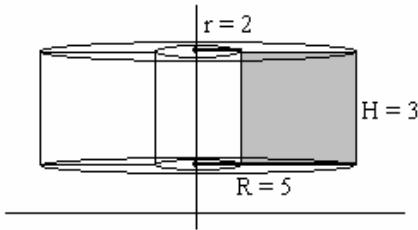


1. $8\left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{1}{2}\right) + (\sqrt{3})^2 = 8\left(\frac{3}{4}\right)\left(\frac{1}{2}\right) + 3 = 3 + 3 = 6$ **ANSWER: 6**

2. $51^2 - 50^2 = 2601 - 2500 = 101$; a = 51, b = 50 **ANSWER: 51**

3. $\sqrt{4x^4} = 4 \quad 4x^4 = 16 \quad x^4 = 4$
 $x = \pm\sqrt[4]{4} = \pm\sqrt[4]{4^2} = \pm\sqrt[4]{2^2} = \pm\sqrt{2}$; other two solutions are imaginary **ANSWER: $\pm\sqrt{2}$**

4.



Revolution creates a cylinder with a cylindrical hole inside

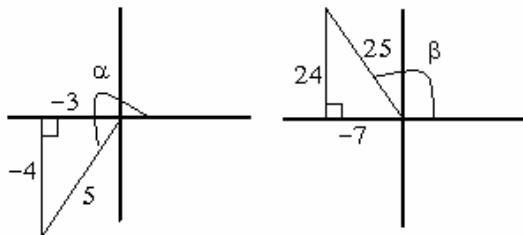
$$V = \pi R^2 H - \pi r^2 H = \pi(5)^2(3) - \pi(2)^2(3) = 75\pi - 12\pi = 63\pi$$
 ANSWER: 63π

5. $(2+2i)^8 = (2\sqrt{2}cis45)^8 = (2\sqrt{2})^8 cis(45 \cdot 8) = (2^8 \cdot 2^4)cis360 = 2^{12}(1) = 2^{12} = 4^6$;
so x = 6 **ANSWER: 6**

6. $f(x) = \frac{2x}{\sqrt{-(x^2 - 7x + 10)}} = \frac{2x}{\sqrt{-(x-2)(x-5)}}$

When $2 < x < 5$, denominator is a non-zero real number, otherwise the function is undefined.
Integers which satisfy are 3 and 4; sum is 7 **ANSWER: 7**

7.



$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta = \left(\frac{-4}{5}\right)\left(\frac{-7}{25}\right) - \left(\frac{-3}{5}\right)\left(\frac{24}{25}\right) = \frac{28 + 72}{125} = \frac{4}{5}$$
 ANSWER: $\frac{4}{5}$



8. When a card is drawn, 47 cards remain, 11 of which match in suit, and 7 of which match in face value

$$P = \frac{7}{47}, Q = \frac{11}{47}, P+Q = \frac{18}{47}$$

ANSWER: $\frac{18}{47}$

9.
$$\lim_{x \rightarrow 4} \left(\frac{2x-8}{x^2-3x-4} + \frac{x-4}{\sqrt{x}-2} \right) = \lim_{x \rightarrow 4} \left(\frac{2(x-4)}{(x+1)(x-4)} + \frac{(\sqrt{x}+2)(\sqrt{x}-2)}{\sqrt{x}-2} \right) = \lim_{x \rightarrow 4} \left(\frac{2}{x+1} + \sqrt{x} + 2 \right)$$

$$= \frac{2}{5} + 2 + 2 = \frac{22}{5}$$

ANSWER: $\frac{22}{5}$ or $4\frac{2}{5}$ or 4.4

10. Multiplying both sides by e^{2x} and simplifying yields the equation

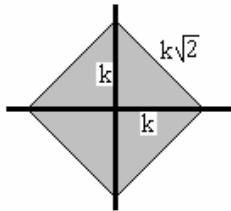
$$2e^{4x} - 17e^{2x} + 30 = 0 \quad (2e^{2x} - 5)(e^{2x} - 6) = 0 \quad e^{2x} = \frac{5}{2} \text{ or } e^{2x} = 6$$

$$2x = \ln \frac{5}{2} \text{ or } 2x = \ln 6$$

$$x = \ln \sqrt{\frac{5}{2}} \text{ or } x = \ln \sqrt{6}$$

$$\text{Sum of solutions} = \ln \sqrt{\frac{5}{2}} + \ln \sqrt{6} = \ln \sqrt{\frac{5}{2} \cdot 6} = \ln \sqrt{15}; \text{ so } p = \sqrt{15} \quad \text{ANSWER: } \sqrt{15}$$

- 11.



Area is a square with side $k\sqrt{2}$, area of square $= (k\sqrt{2})^2 = 2k^2$ **ANSWER:** $2k^2$

12.
$$\frac{A}{x-2} + \frac{B}{x-4} = \frac{8x-26}{x^2-6x+8}$$

$$\frac{A(x-4) + B(x-2)}{x^2-6x+8} = \frac{8x-26}{x^2-6x+8}$$

$$Ax - 4A + Bx - 2B = 8x - 26$$

$$Ax + Bx = 8x \text{ and } -4A - 2B = -26$$

Solve system of equations to find A = 5, B = 3 $A^B = 5^3 = 125$ **ANSWER:** 125