1

1. 1
2. 7
3. $\frac{2750\pi}{3}+3000$
4. 172π
5. $\frac{10}{\pi}-2$
6. $\frac{1+\sqrt{5}}{2}$
7. $\frac{20\sqrt{3}}{9}$
8. $18\pi-36$
9. $2\sqrt{2}-2\sqrt{6}+4$
10. $\frac{28\pi}{3}-8\sqrt{3}+8$
11. 12
12. $3\sqrt{3}x$
13. 120
14. 7
15. 252π
16. $18\sqrt{3}$
17. 16
18. $18\sqrt{3}$
19. 6
20. 10
21. 2
22. $123\frac{5}{6}^\circ$ or $123.8\overline{3}^\circ$
23. Frustum
24. 5
25. 3

1. 1

(area of BDFH) – (area of ABH + area of CDB + area of EFD + area of GHF)

25 – (6 + 6 + 6 + 6) = 1

2. 7

243π (Cylinder volume) - $\frac{243\pi}{3}$ (cone volume) = $\frac{2}{3}(243\pi)$

Hemisphere volume = $\frac{2}{3}(\pi r^3)$ = $\frac{2}{3}(243\pi)$ $\therefore r = 7$

3. $\frac{2750\pi}{3}+3000$

Roaming space comprises: 12 quarter-cylinders of radius 5 and height 10 (1 at each edge)
8 eighths of a sphere of radius 5 (one at each vertex)
Six 10x6x5 rectangular prisms (one for each face)

Adding up volumes: $12(\frac{1}{4}\pi\cdot 5^2\cdot 10)+8(\frac{1}{8}\cdot\frac{4}{3}\cdot\pi\cdot 5^3)+6(10\cdot 10\cdot 5)=\frac{2750\pi}{3}+3000$

4. 172π

Volume of a frustum = $\frac{\pi r^2+\pi R^2+\pi rR}{3}\cdot h$

r = 5, R = 8, h = 4 $\therefore$ V = 172π

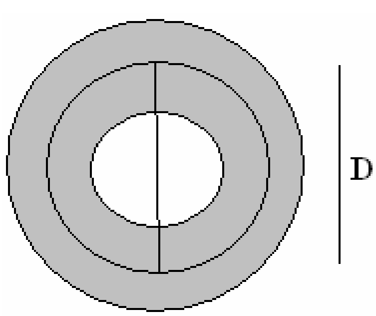
5. $\frac{10}{\pi}-2$

The central axis will define a circle with circumference equal to the original height of the cylinder.

Height = 10 = πD $\therefore$ D= $\frac{10}{\pi}$

The hole in the middle will have diameter = D - (diameter of cylinder)

= $\frac{10}{\pi}-2$



6. $\dfrac{1+\sqrt{5}}{2}$ The golden ratio, φ.

7. $\dfrac{20\sqrt{3}}{9}$

$\overline{JI} = \overline{IM} = \overline{MJ} = 4$. Triangles RIA, AMD, and DJR are 30-60-90

$$\therefore \overline{IA} = \overline{MD} = \overline{JR} = \frac{2}{3}\cdot 4 = \frac{8}{3} \text{ and } \overline{RI} = \overline{AM} = \overline{DJ} = \frac{1}{3}\cdot 4 = \frac{4}{3}.$$

$$\overline{RA} = \sqrt{\overline{IA}^2 - \overline{RI}^2} = \frac{4\sqrt{5}}{3}$$

$$\text{Area of RAD} = \frac{\overline{RA}^2\sqrt{3}}{4} = \left(\frac{4\sqrt{5}}{3}\right)^2 \cdot \frac{\sqrt{3}}{4} = \frac{20\sqrt{3}}{9}$$

8. $18\pi - 36$

Let M be the midpoint of $\overline{AB}$. AM=AB=QM=6. Radius $= 6\sqrt{2}$.
Sector-triangle=segment= ¼ (72π) – ½ (12)(6) = 18π - 36

9. $2\sqrt{2} - 2\sqrt{6} + 4$

$$XY = 2\sqrt{2} + 2\sqrt{6}$$

$$YP = 2\sqrt{2} + 2\sqrt{6} - 4$$

$$XQ = 2\sqrt{2} + 2\sqrt{6} - 4\sqrt{2} = 2\sqrt{6} - 2\sqrt{2}$$

$$PQ = XY - \left(YP + XQ\right) = 2\sqrt{2} + 2\sqrt{6} - (2\sqrt{2} + 2\sqrt{6} - 4 + 2\sqrt{6} - 2\sqrt{2})$$

$$= 2\sqrt{2} - 2\sqrt{6} + 4$$

10. $\dfrac{28\pi}{3} - 8\sqrt{3} - 8$

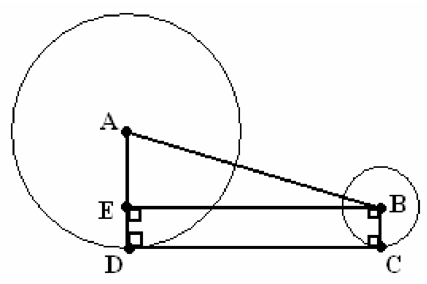
Segment area=sector area-triangle area

$$\text{MQ}\overline{\text{NRM}} = \frac{1}{6}(4\sqrt{2})^2\pi - (4\sqrt{2})^2\frac{\sqrt{3}}{4} = \frac{16\pi}{3} - 8\sqrt{3}$$

$$\text{MP}\overline{\text{NRM}} = \frac{1}{4}(16\pi) - 4\cdot 4\cdot\frac{1}{2} = 4\pi - 8$$

$$(\frac{16\pi}{3} - 8\sqrt{3}) + (4\pi - 8) = \frac{28\pi}{3} - 8\sqrt{3} - 8$$

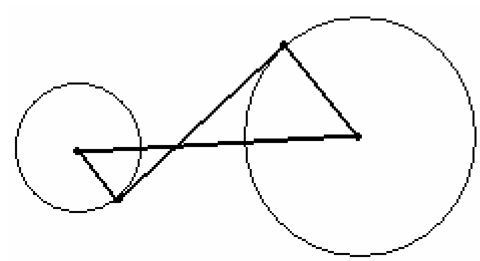
11. 12



BE is parallel to DC and their lengths are equal. ED and BC are both also equal lengths.
This information yields ABE as a 5-12-13 right triangle.

12. $3\sqrt{3}x$

The triangles formed are similar by AAA. The radii show they are similar in a 2:1 ratio, therefore they divide the segment between their centers into a 2:1 ratio.
Two right triangles are formed of side lengths $x, \sqrt{3}x, 2x$ and $2x, 2\sqrt{3}x, 4x$. $\sqrt{3}x + 2\sqrt{3}x = 3\sqrt{3}x$

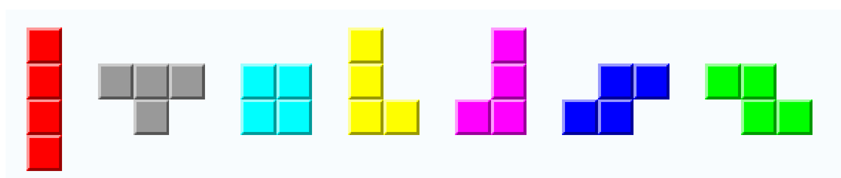


13. 120

p=4 (tetrahedron), q=6 (cube), r=8 (octahedron), s=12 (dodecahedron), t=20 (icosahedron).

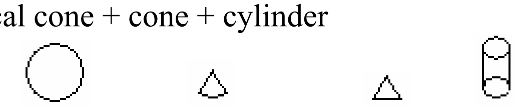
14. 7

They are the pieces used in Tetris.



15. 252π

Volume = sphere – spherical cone + cone + cylinder

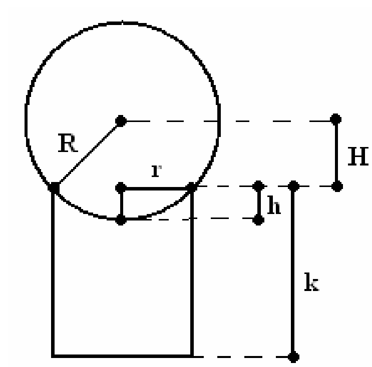


$$= \frac{4}{3}\pi R^3 \quad - \quad \frac{2}{3}\pi R^2 h \quad + \quad \frac{1}{3}\pi r^2 H + \pi r^2 k$$

$$= \frac{4}{3}\pi \cdot 25 \quad - \quad \frac{2}{3}\pi \cdot 25 \quad + \frac{1}{3}\pi \cdot 9 \cdot 4 + \pi \cdot 9 \cdot 10$$

$$= \frac{500}{3}\pi \quad - \quad \frac{50}{3}\pi \quad + \ 12\pi + 90\pi$$

$$= 252\pi$$



16. $18\sqrt{3}$

The short diagonal of the small hexagon is the same length as the side of the larger hexagon: 6. right triangles yield the side of the smaller hexagon to be $2\sqrt{3}$. The hexagon's

area= $(2\sqrt{3})^2 \cdot \frac{3\sqrt{3}}{2} = 18\sqrt{3}$

17. 16

The reflection forms two similar triangles, one 3-4-5 and one 9-12-15. The legs of length 4 and 12 stretch the length of the puddle.

18. $18\sqrt{3}$

This forms an equilateral triangle whose sides coincide with the diagonals of three faces of the cube.

$$\frac{(6\sqrt{2})^2\sqrt{3}}{4} = 18\sqrt{3}$$

19. 6

(Sphere's volume)(Sphere's distance to fulcrum) = (Cube's volume)(Cube's distance to fulcrum)

$\pi^3 \cdot \pi = \frac{4\pi}{3}\left(\frac{\pi}{2}\right)^3 \cdot x$ Solving out yields x=6

20. 10

21. 2

In a square it is $\sqrt{2}$, in a cube it is $\sqrt{3}$, in a hypercube it is $\sqrt{4} = 2$.

22. $123\frac{5}{6}°$ or $123.8\overline{3}°$

The hour hand moves at a rate of $0.5°$/minute. The minute hand moves at a rate of $6°$/minute. Find the position in relation to 12:00 of each hand, and then find the difference in these positions to get the angle between the hands at exactly 7:15:40 am.

hour hand position is $\frac{0.5°}{\text{min}} \cdot (7 \cdot 60 + 15 + 2/3)\,\text{min} = 217\tfrac{5}{6}°$

minute hand position is $\frac{6°}{\text{min}} \cdot (15 + 2/3)\,\text{min} = 94°$

difference is $217\tfrac{5}{6}° - 94° = 123\tfrac{5}{6}°$

23. Frustum

24. 5

9-12-15, 15-20-25, 8-15-17, 15-36-39, 15-112-113

25. 3

Continuing this shape into a full circle with 6 smaller circles inscribed reflects the ability of circles to being packed hexagonally. The space in the middle of the smaller circles can be filled by another unit circle that is tangent to the other 6. Connecting three collinear diameters of these smaller circles will create a diameter for the larger circle: 6. Thus, the radius of the larger circle is 3.