Mu Alpha Theta
2006 National Convention

Hustle
Pre Calculus & Calculus
Answers

1) $-\frac{\sqrt{2}}{2}$

2) $\frac{3}{5}$ (or 0.6)

3) $15\sqrt{6}$

4) $ac+bc$ (or $c(a+b)$)

5) 324

6) 207

7) $\frac{2}{3}$ (or $0.\overline{6}$)

8) $\frac{7}{16}$ (or 0.4375)

9) $\frac{18}{19}$

10) $\frac{\sqrt{5}}{5}$

11) $\frac{\sqrt{6}-\sqrt{2}}{2}$

12) 8

13) 0

14) $20^\circ 16' 12''$

15) $\frac{77\pi}{90}$

16) 3720π (yards per hour)

17) $\frac{128\sqrt{3}}{9}$

18) $-\frac{16}{7}$

19) -20

20) $-\frac{42}{5}$ (or -8.4)

21) 4

22) $-\frac{1}{8}$

23) 1

24) $\frac{68}{3}$

25) $10+5\sqrt{3}$

1. Since the cosine is squared, look for the θ with the negative sine – only $\frac{5\pi}{4}$ fits.

$$f\left(\frac{5\pi}{4}\right)=2\left(-\frac{\sqrt{2}}{2}\right)^2\left(-\frac{\sqrt{2}}{2}\right)=-\frac{\sqrt{2}}{2}$$

2. The linear factor is $(x-5)$. Work backwards to find the quadratic factor.

$x=i-2$

$x+2=i$

$(x+2)^2=-1$

$x^2+4x+5=0$

So $p(x)=-(x^2+4x+5)(x-5)$ ….the leading negative because the graph rises to the left, so it must fall to the right, being an odd degree.

Expanding, $p(x)=-x^3+x^2+15x+25$,

and then $\frac{a+b+c}{d}=\frac{15}{25}=\frac{3}{5}$.

3. $\cos^2(B)=\frac{5}{8}$, so $\cos B=\frac{\sqrt{5}}{\sqrt{8}}$ and using the Pythag. Theorem, $\sin B=\frac{\sqrt{3}}{\sqrt{8}}=\frac{\sqrt{3}}{2\sqrt{2}}$.

The area of a triangle is half the product of any two sides and the sine of their included angle.

$$A=\frac{1}{2}(12)(10)\frac{\sqrt{3}}{2\sqrt{2}}=\frac{30\sqrt{3}}{\sqrt{2}}=15\sqrt{6}.$$

4. $\ln(6)=\frac{\log 6}{\log e}=\frac{\log 2+\log 3}{\log e}=\frac{a+b}{\log e}$.

Now, note that $\log e=\frac{\ln e}{\ln 10}=\frac{1}{c}$.

So $\ln(6)=\frac{a+b}{\frac{1}{c}}=c(a+b)$.

5. This is a logistics growth model. The least value occurs when $t=0$.

$$P=\frac{36}{1+3e^0}=\frac{36}{4}=9.$$

As t gets very large, e^{-6t} approaches 0, yielding a maximum of $\frac{36}{1+0}=36$.

Multiply: $(9)(36)=324$.

6. To find the cross-product, use the matrix determinant shortcut.

$$\vec{W}\times\vec{X} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 2 & 5 \\ 1 & 4 & -2 \end{vmatrix} = \begin{matrix}(-4\vec{i}+5\vec{j}-12\vec{k})- \\ (20\vec{i}+6\vec{j}+2\vec{k})\end{matrix} = -24\vec{i}-\vec{j}-14\vec{k}.$$

To find the dot product, add the products of the corresponding components.

$$\left(-24\vec{i}-\vec{j}-14\vec{k}\right)\bullet\left(-10\vec{i}+5\vec{j}+2\vec{k}\right) = \quad 240-5-28=207.$$

7. Solve for *r*, and put the equation into standard polar form. The eccentricity is then the coefficient of $\cos\theta$.

$$2r\cos\theta+3r=24$$
$$r(2\cos\theta+3)=24$$
$$r=\frac{24}{3+2\cos\theta}$$
$$r=\frac{24}{3\left(1+\frac{2}{3}\cos\theta\right)}$$
$$r=\frac{8}{1+\frac{2}{3}\cos\theta}$$

8. The first set can be represented by a circle in the complex plane, centered at zero with radius four. The second set is a circle of radius 3 at the same center. The area between the circles is $16\pi-9\pi=7\pi$, and the probability of a point in the large circle being in this annulus is $\frac{7\pi}{16\pi}=\frac{7}{16}$.

9. This is a rational expression with a removable discontinuity at $x=0.\overline{6}=\frac{2}{3}$.

$$\frac{27x^3-8}{21x^2+10x-16}=\frac{(3x-2)(9x^2+6x+4)}{(3x-2)(7x+8)}=$$
$$\frac{(9x^2+6x+4)}{(7x+8)}.$$

Now, evaluate this at $x=\frac{2}{3}$.

$$\frac{9\left(\frac{2}{3}\right)^2+6\left(\frac{2}{3}\right)+4}{7\left(\frac{2}{3}\right)+8}=\frac{4+4+4}{\frac{14}{3}+\frac{24}{3}}=\frac{12}{\frac{38}{3}}=\frac{36}{38}=\frac{18}{19}$$

10. Substitute $A = x^2$, and then:

$$5A^2 + 4A - 1 = 0$$
$$(5A-1)(A+1) = 0$$
$$A = \frac{1}{5} \;\; or \;\; -1$$

since $x = \pm\sqrt{A}$, only the first value of *A* yields real values of *x*. The greater value is the positive one: $\sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5}$.

11. Note that $\cos E = \cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right) =$

$$\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4} =$$
$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = .$$
$$\frac{(\sqrt{3})(\sqrt{2})}{2} - \frac{(1)(\sqrt{2})}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}.$$

(of course, you probably had that memorized, after a whole season of competing!)

$\cos E = \frac{DE}{EF}$, so $\frac{\sqrt{6}-\sqrt{2}}{4} = \frac{x}{2}$, and then $x = \frac{\sqrt{6}-\sqrt{2}}{2}$.

12. The circle's area is 5π, so its radius is $\sqrt{5}$. It is centered at the rectangular point $(2\sqrt{2}\cos(-45^\circ),\ 2\sqrt{2}\sin(-45^\circ))$, better known as the point $(2,-2)$. Manipulating the equation of the circle in standard form:

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-2)^2 + (y+2)^2 = 5$$
$$x^2 - 4x + 4 + y^2 + 4y + 4 = 5$$
$$x^2 - 4x + y^2 + 4y + 3 = 0$$

And then $B + DE = (-4) + (4)(3) = 8$.

13. Let's plug in $x+\frac{\pi}{6}$ to find $f(x)$ first.

$$f(x)=f\left(\left(x+\frac{\pi}{6}\right)-\frac{\pi}{6}\right)$$
$$f(x)=4\sin\left(3\left(x+\frac{\pi}{6}\right)+\frac{\pi}{2}\right)$$
$$f(x)=4\sin\left(3x+\frac{\pi}{2}+\frac{\pi}{2}\right)$$
$$f(x)=4\sin(3x+\pi)$$

And then…..

$$g\left(\frac{4\pi}{3}\right)=f\left(\left|\frac{4\pi}{3}\right|+\frac{\pi}{3}\right)=f\left(\frac{5\pi}{3}\right)=$$
$$4\sin\left(3\left(\frac{5\pi}{3}\right)+\pi\right)=4\sin(5\pi+\pi)=.$$
$$4\sin(6\pi)=4(0)=0.$$

14. Keep multiplying the part after the decimal by 60.

Of course, $D = 20$.

$(0.27)(60)=16.2$, so $M = 16$.

$(0.2)(60)=12$, so $S = 12$.

So, $20.27^\circ = 20^\circ 16'12''$.

15. Simply keep adding 360 until you get the positive angle 154°. Convert it to radians.

$$154^\circ=\left(\frac{154^\circ}{180^\circ}\right)\left(\frac{\pi}{1}\right)=\frac{77\pi}{90}.$$

16. Mind your unit conversions! (Remember, radians are unitless.) Find the angular velocity first:

$$\omega=\left(\frac{2\pi}{revolution}\right)\left(\frac{31\ revolutions}{2\min}\right)\left(\frac{60\min}{hour}\right)$$

Linear speed is found using the formula

$$\omega=\frac{1860\pi}{hour}$$

$$v=r\omega$$
$$v=(2\ yards)\left(\frac{186\pi}{hour}\right)=3720\pi\ \frac{yards}{hour}.$$

17. The area of a regular hexagon with side lengths x is given by $A=\frac{3}{2}x^2\sqrt{3}$. Since $x=\frac{P}{6}$, where P is perimeter, $A=\frac{3}{2}\left(\frac{P}{6}\right)^2\sqrt{3}=\frac{P^2\sqrt{3}}{24}$.

Plugging in the given numbers confirms the existence of the anticipated infinite geometric series:

$$\frac{32\sqrt{3}}{3}+\frac{8\sqrt{3}}{3}+\frac{2\sqrt{3}}{3}+\frac{0.5\sqrt{3}}{3}+....$$

The sum is $\frac{a_o}{1-r}=\frac{\frac{32\sqrt{3}}{3}}{1-\frac{1}{4}}=\frac{\frac{32\sqrt{3}}{3}}{\frac{3}{4}}=\frac{128\sqrt{3}}{9}$

18. Solve for the inverse.

$$x=\frac{3y-1}{2y+3}$$
$$2xy+3x=3y-1$$
$$3y-2xy=3x+1$$
$$y(3-2x)=3x+1$$
$$f^{-1}(x)=y=\frac{3x+1}{3-2x}$$

Then, $f^{-1}(5)=\frac{3(5)+1}{3-2(5)}=-\frac{16}{7}$.

19. **A:** For even functions, $f(-x)=f(x)$,
so $f(-2)=f(2)=-2$

B: For odd functions, $f(-x)=-f(x)$,
so $f(-3)=-f(3)=-2$

C: $f^{-1}\left(-\left(\left|-2\right|\right)\right)=f^{-1}(-2)$. Since the function is one-to-one, the x-value in the table which yields -2 is the only possible answer. Since $(2,-2)$ lies on the graph of $f(x)$, $(-2,2)$ lies on the graph of $f^{-1}(x)$.

$A+4B-5C=-2+4(-2)-5(2)=-20$.

20. Vectors are orthogonal iff their dot product is zero. Solve $-k+12=0$, and $k=12$.

Vectors that are parallel have corresponding components in a constant ratio. Solve $\frac{-2}{m}=\frac{7}{5}$, and $m=-\frac{10}{7}$.

$\frac{k}{m}=\frac{12}{-\frac{10}{7}}=-\frac{42}{5}$.

21. $f'(\theta)=\sec^2\theta$; $f''(\theta)=2\left(\sec^2\theta\right)(\tan\theta)$. $2\sec^2\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{4}\right)=2(2)(1)=4$

22. This is the definition of the derivative, $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, when $f(x)=\frac{1}{2x}$, evaluated at $x=2$. $f'(x)=-\frac{1}{2x^2}$, so $f'(2)=-\frac{1}{8}$.

23. Memorized?

24. $\left.\frac{2}{3}x^3+x\right|_{-1}^{3}=(18+3)-\left(-\frac{2}{3}-1\right)=\frac{68}{3}$

25. The area of the triangle is $A=\frac{x^2\sin\theta}{2}$

Differentiating with respect to time, $\frac{dA}{dt}=\frac{1}{2}\left(2x\frac{dx}{dt}\sin(\theta)+x^2\frac{d\theta}{dt}\cos(\theta)\right)$.

Substituting, $\frac{dA}{dt}=\frac{1}{2}\left(2(10)(2)\left(\frac{1}{2}\right)+(100)\left(\frac{1}{5}\right)\left(\frac{\sqrt{3}}{2}\right)\right)$

$=10+5\sqrt{3}$