1. A
2. E
3. D
4. B
5. D
6. A
7. C
8. C
9. D
10. E
11. C
12. A
13. A
14. D
15. C
16. C
17. D
18. A
19. B
20. D
21. B
22. D
23. B
24. D
25. C
26. A
27. C
28. E
29. C
30. B

1: Marginal Cost =
$2+0.002x = 2+0.002(1000) = 4 \therefore A$

2: Velocity is decreasing when acceleration is negative.
Position = $t^3-6t^2+12t-8$
Velocity = $3t^2-12t+12$
Acceleration $=6t-12=0$, $t=2$
The acceleration is negative when $t<2$ or
$(-\infty,2) \therefore E$

$3: x = 3y - y^2$

$dx/dt = 3dy/dt - 2ydy/dt$

$dx/dt = 3(3)-2(1)(3) = 3$

$speed = \sqrt{(dx/dt)^2+(dy/dt)^2}$

$= \sqrt{(3^2+3^2)} = 3\sqrt{2} \therefore D$

$4: y^3 - xy^2 = 4 \to 2^3 - x(2^2) = 4 \to$

$8-4x = 4 \to x = 1$

$3y^2y' - y^2 - 2xyy' = 0$

$y'(3y^2-2xy) = y^2$

$y' = \dfrac{y^2}{3y^2-2xy} = \dfrac{2^2}{3(2)^2-2(1)(2)} = 1/2 \therefore B$

$5: P(t) = 3t\hat{i} + e^t\hat{j} \to v(t) = 3\hat{i} + e^t\hat{j} \to a(t) = e^t\hat{j}$
Acceleration varies in magnitude with time but is always in the positive j direction. $\therefore$ D is the correct answer.

6: The 3rd order Maclaurin series for sin(x) and cos(x) are $\sin(x) = x - \dfrac{x^3}{3!}$ and $\cos(x) = 1 - \dfrac{x^2}{2!}$.

$\sin 3 - \cos 3 = 3 - \dfrac{3^3}{3!} - 1 + \dfrac{3^2}{2!} = 3 - \dfrac{27}{6} - 1 + \dfrac{9}{2}$

$= 2 - \dfrac{9}{2} + \dfrac{9}{2} = 2 \therefore A$

$7: f(x) = e^{-x/2}, f'(x) = -\dfrac{e^{-x/2}}{2},$

$f''(x) = \dfrac{e^{-x/2}}{4}, f'''(x) = -\dfrac{e^{-x/2}}{8}, f'''(0) = -\dfrac{1}{8}$

$coefficient = \dfrac{f'''(0)}{3!} = -\dfrac{1}{8(6)} = -\dfrac{1}{48} \therefore C$

$8: A = \pi r^2, Average = \dfrac{1}{5-2}\int_2^5 \pi r^2 dr = \dfrac{\pi}{3}\dfrac{r^3}{3}\Big|_2^5$

$= \dfrac{125\pi}{9} - \dfrac{8\pi}{9} = \dfrac{117\pi}{9} = 13\pi \therefore C$

$9: \lim_{x\to 1} \arcsin\left(\dfrac{1-\sqrt{x}}{1-x}\right) = \arcsin \lim_{x\to 1}\left(\dfrac{1-\sqrt{x}}{1-x}\right)$

$= \arcsin \lim_{x\to 1} \dfrac{1-\sqrt{x}}{(1+\sqrt{x})(1-\sqrt{x})} = \arcsin \lim_{x\to 1} \dfrac{1}{(1+\sqrt{x})}$

$= \arcsin(1/2) = \pi/6 \therefore D$

$10: \int_{\ln 3}^{\ln 5} \dfrac{e^{3x-3}}{e^{x-4}} dx = \int_{\ln 3}^{\ln 5} e^{2x+1} dx = \dfrac{e^{2x+1}}{2}\Bigg]_{\ln 3}^{\ln 5}$

$= \dfrac{1}{2}(e^{2\ln 5+1} - e^{2\ln 3+1}) = \dfrac{1}{2}e(25-9) = 8e \therefore E$

$11: a = \sqrt{y}, b = \sqrt{x} \to a^2 = b^2 - b^4$

$2a(da/db) = 2b - 4b^3$

$da/db = d(y^{1/2})/d(x^{1/2})$

$= \dfrac{2b-4b^3}{2a} = \dfrac{2\sqrt{x} - 4x\sqrt{x}}{2\sqrt{x-x^2}}$

$= \dfrac{\sqrt{x}(1-2x)}{\sqrt{x-x^2}} \therefore C$

$12: Diagonal = \sqrt{s^2+s^2+s^2} = s\sqrt{3}$

$dD/dt = \sqrt{3}(ds/dt) = 5$

$ds/dt = 5/\sqrt{3}$

$V = s^3 \to dV/dt = 3s^2 ds/dt = 3(2)^2(5/\sqrt{3})$

$= 60/\sqrt{3} = 20\sqrt{3} \therefore A$

$$13: \int_0^{\ln 2} (\sinh x)dx = \int_0^{\ln 2} \frac{e^x - e^{-x}}{2} dx = \frac{e^x + e^{-x}}{2}\Big|_0^{\ln 2}$$

$$= \frac{e^{\ln 2} + e^{-\ln 2}}{2} - \frac{e^0 + e^{-0}}{2} = \frac{2 + 1/2}{2} - \frac{1+1}{2}$$

$$= \frac{5}{4} - 1 = 1/4 \therefore A$$

$$14: \ln y = x^2 \ln x$$

$$\frac{y'}{y} = 2x \ln x + x$$

$$y' = y(2x \ln x + x) = 2^4(4\ln 2 + 2)$$

$$= 64 \ln 2 + 32 \therefore D$$

$$15: \lim_{x \to 0} \frac{\sqrt{6+2x} - \sqrt{6+x^2}}{\sqrt{3+4x} - \sqrt{3-x^3}} \xrightarrow{L'Hospitals}$$

$$\lim_{x \to 0} \frac{(6+2x)^{-1/2} - x(6+x^2)^{-1/2}}{2(3+4x)^{-1/2} - (3x^2/2)(3-x^3)^{-1/2}}$$

$$= \frac{6^{-1/2}}{2(3)^{-1/2}} = \frac{\sqrt{3}}{2\sqrt{6}} = \frac{\sqrt{2}}{4} \therefore C$$

$$16: \int_{-\pi/4}^{\pi/4} (1 + \tan^2 x)dx = \int_{-\pi/4}^{\pi/4} (\sec^2 x)dx = \tan x \Big|_{-\pi/4}^{\pi/4}$$

$$= \tan(\pi/4) - \tan(-\pi/4) = 1 - -1 = 2 \therefore C$$

$$17: f'(x) = 5\cos x - \sqrt{3}\sin x = 0 \to \tan x = \frac{5}{\sqrt{3}}$$

By drawing an appropriate right triangle using the Pythagorean theorem, one can evaluate

$$\sin x = \frac{5}{\sqrt{28}}, \cos x = \frac{\sqrt{3}}{\sqrt{28}}$$

$\therefore$ maximum value of $y = \sqrt{3}(\frac{\sqrt{3}}{\sqrt{28}}) + 5(\frac{5}{\sqrt{28}})$

$$= \frac{28}{\sqrt{28}} = 2\sqrt{7} \therefore D$$

$$19: x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{-1}{1} = 2$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{3}{8} = 13/8 \therefore B$$

$$20: \lim_{x \to 0^+} \frac{1}{2 + 10^{1/x}} = \frac{1}{2 + 10^{\infty}} = 0$$

$$\lim_{x \to 0^-} \frac{1}{2 + 10^{1/x}} = \frac{1}{2 + 10^{-\infty}} = 1/2$$

$$\lim_{x \to 0^+} y \neq \lim_{x \to 0^-} y$$

$\therefore$ Limit does not exist, D.

$$18: mass = \int_0^L x^4 dx = \frac{x^5}{5}\Big|_0^L = \frac{L^5}{5}$$

$$C.M. = \frac{5}{L^5}\int_0^L x^5 dx = \frac{5}{L^5}\frac{x^6}{6}\Big|_0^L = \frac{5}{L^5}(\frac{L^6}{6}) = \frac{5L}{6} \therefore A$$

21: Using Integration by Parts:

$$\int x^2 \cos x dx = f(x) - \int 2x \sin x dx$$

$$x^2 \sin x + 2x \cos x - 2\sin x = f(x) - (-2x\cos x + 2\sin x)$$

$$f(x) = x^2 \sin x + 2x\cos x - 2\sin x - 2x\cos x + 2\sin x$$

$$= x^2 \sin x + C \therefore B$$

$$22: \tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{3b - b}{1 + 3b^2} = \frac{2b}{1 + 3b^2},$$

$$\theta = \tan^{-1}(\frac{2b}{1+3b^2})$$

$$\theta' = \frac{\dfrac{2(1+3b^2) - 6b(2b)}{(1+3b^2)^2}}{1 + (\dfrac{2b}{1+3b^2})^2} = 0 \to 2(1+3b^2) - 6b(2b) = 0$$

$$2 + 6b^2 - 12b^2 = 0 \to 2 = 6b^2 \to b = \sqrt{3}/3 \therefore D$$

23 : $x = 2\sin\theta \rightarrow dx = 2\cos\theta d\theta$

$$\int \frac{x^2}{\sqrt{4-x^2}}dx = \int \frac{4\sin^2\theta}{\sqrt{4-4\sin^2\theta}} 2\cos\theta d\theta$$

$$= \int \frac{4\sin^2\theta}{2\cos\theta} 2\cos\theta d\theta = \int 4\sin^2\theta d\theta$$

$$= 2\int (1-\cos 2\theta)d\theta = 2\theta - \sin 2\theta$$

$$= 2\theta - 2\sin\theta\cos\theta$$

$$= 2\sin^{-1}(x/2) - 2(\frac{x}{2})(\frac{\sqrt{4-x^2}}{2})$$

$$= 2\sin^{-1}\frac{x}{2} - \frac{x\sqrt{4-x^2}}{2} + C \therefore B$$

$$24: \lim_{n\to\infty}\sum_{k=1}^{n}\frac{2n^2+2kn+k^2}{n^3}$$

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}(\frac{2n^2+2kn+k^2}{n^2})$$

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}(2+2(\frac{k}{n})+(\frac{k}{n})^2)$$

$$= \int_0^1 (2+2x+x^2)dx = 2x+x^2+\frac{x^3}{3}\Big|_0^1 = 10/3 \therefore D$$

$$25: dx/dt = 2(-\sin t + \sin t + t\cos t) = 2t\cos t$$

$$dy/dt = 2(\cos t - \cos t + t\sin t) = 2t\sin t$$

$$arclength = \int_0^\pi \sqrt{(2t\cos t)^2 + (2t\sin t)^2}dt$$

$$= \int_0^\pi \sqrt{4t^2\cos^2 t + 4t^2\sin^2 t}dt = \int_0^\pi \sqrt{4t^2}dt = \int_0^\pi 2tdt = t^2\Big|_0^\pi$$

$$= \pi^2 \therefore C$$

$$26: curvature = \frac{y''}{[1+(y')^2]^{3/2}}$$

$$y' = 3x^2, y'(1) = 3, y'' = 6x, y''(1) = 6$$

$$curvature = \frac{6}{(1+3^2)^{3/2}} = \frac{6}{10^{3/2}}$$

$$radius = 1/curvature = \frac{10^{3/2}}{6} \therefore A$$

$$27: y = 1 + \cfrac{x}{2 + \cfrac{x}{1 + \cfrac{x}{2 + \cfrac{x}{1 + \ldots}}}} = 1 + \cfrac{x}{2+\cfrac{x}{y}} = 1 + \cfrac{x}{\cfrac{2y+x}{y}}$$

$$y = 1 + \frac{xy}{2y+x} = \frac{2y+x+xy}{2y+x} \rightarrow 2y^2 + xy = 2y + x + xy \rightarrow$$

$$2y^2 - 2y - x = 0$$

$$y(4): 2y^2 - 2y - 4 = 0 \rightarrow y = 2$$

$$4y(y') - 2(y') - 1 = 0 \rightarrow 4(2)(dy/dx) - 2(dy/dx) - 1 = 0$$

$$dy/dx = 1/6 \therefore C$$

$$28: \frac{A}{9-B^2} \le 1 \rightarrow A \le 9 - B^2$$

By graphing the function A = 9 – B^2 in the window [0, 5] X [0, 10], one can see that the probability is equal to the area under the curve divided by the total area of the specified range and domain.

$$probability = \frac{\int_0^3 (9-B^2)dB}{5*10} = \frac{9B - B^3/3\Big|_0^3}{50}$$

$$= \frac{9(3) - \frac{3^3}{3}}{50} = \frac{27-9}{50} = 18/50 = 0.36 \therefore A$$

$$29: SA = 2\pi\int_a^b y\sqrt{1+(dy/dx)^2}dx$$

$$dy/dx = 2x+2$$

$$SA = 2\pi\int_1^3 (x^2+2x+2)\sqrt{1+(2x+2)^2}dx$$

$$= 2\pi\int_1^3 (x^2+2x+2)\sqrt{4x^2+8x+5}dx \therefore C$$

30 : By the theorem of Pappus, the volume of the solid is given by V = Ad, where A is the area of the ellipse and d is the distance travelled by centroid of the ellipse during 1 revolution. The area of the ellipse is A = πab and the distance d is given by d = $2\pi r$, where r is the distance from the centroid of the ellipse (0, 0) to the line 3x + 4y = 25, which can be found using the point to line distance formula. Thus, $V = \pi ab * 2\pi r$.

$$r = \frac{|3(0) + 4(0) - 25|}{\sqrt{3^2 + 4^2}} = 5$$

$$\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1 \rightarrow A = \pi ab = \pi(4)(3) = 12\pi$$

$$V = 12\pi * 2\pi(5) = 120\pi^2 \therefore B$$