Please skip to the directions below and do not take the time to read the following paragraph until you have finished the test. This paragraph is the only part of the test you do not need to read in order to complete it. My name is Mr. Snube, and I am an adventurer and high school math enthusiast. A year ago, I was on duty in Hawaii, exploring uninhabited portions of the Island (caves, mountains, volcanoes, lagoons; the usual). I happened upon the ruins of an ancient Hawaiian temple, and, being the adventurer I am, I decided to investigate. The center piece of the temple's ruins was a large altar made entirely from obsidian, a black volcanic glass. The top of the altar had a small compartment which I opened up, expecting to find treasure. Instead, all I got was problems. These problems, to be exact. I worked on them for hours and couldn't solve them, so I compiled them for all of you to try. I hope you have better luck than I had with this highly mysterious test.

Directions:

- First and foremost, always follow directions. Second, you are not expected to answer all of the questions, so skip whenever necessary and then go back. If you get bogged down by one or two problems and run out of time, well, just don't say I didn't warn you. Thirdly, if you ever come to a question that has none of the correct answers as a choice, or has a serious flaw which yields it unanswerable, bubble NOTA. If more than one of the answer choices are equivalent to the correct answer, bubble one of them and write a dispute; do NOT bubble NOTA. If a problem does not exist, and is labeled as such, then bubble D. Good luck.
- 1. The answer to number one is 13. Please go on to number two.
 - A. 2 B. 3 C. 5 D. 13 E. 21
- 2. Find the difference between the smallest perfect square greater than one million and the largest perfect square less than one million.
 - A. 2000 B. 2001 C. 2006 D. 4000 E. NOTA
- 3. In order to open the compartment housing the test, I, *Mr. Snube*, had to unlock it. In order to unlock the compartment, I had to list a set containing at least one number from each of the following categories:

complex numbers	integers	irrational numbers	natural numbers
rational numbers	real numbers	transcendental	
		numbers	

I completed this task using the fewest possible numbers. How many numbers were in my set?

A. 1 B. 2 C. 3 D. 4 E. NOTA

4. Please go to question 12. Do not complete question four at this time.

A.Yes B. No C. NO)TA
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5. If you are following directions and not skipping ahead from number four, then you should have about 12 minutes left. In celebration of this, you may now return to question 12 if you have gained enough information to answer it, *but please come back to question five* when you are done doing what you need to do. There is an answer to number five, but it is hidden.

A. DAVINCI B. LETTERS C. NUMBERS D. PERFECT E. MYSTERY

6. What is the relationship between the two rows of numbers?

18, 38, 24, 46, 42

8, 24, 8, 24, 8

A. Each term of the second row equals the product of the digits of the corresponding term in the first row

B. Read answer choice D but don't bubble D or B for this question.

C. There is no relationship between the two rows of numbers.

- D. An *ambigram* is a number that looks the same when the paper it is written on is turned upside-down. E. NOTA
- 7. In the sequence of numbers 2, 5, 3, ... each term (starting with the third) is equal to the term preceding it minus the term preceding that one. What is the sum of the first 100 terms of this sequence?
 - A. 2 B. -2 C. 10 D. -8 E) NOTA
- 8. Find the greatest number of sides a regular polygon can have while still having a whole number for its interior angle measure.
 - A. 180 B. 720 C. 360 D. No upper limit E. NOTA
- 9. Find the missing number in the table:

		8	20	28		
		15	21	45		
		17	29	?	Ī	
					-	
A. 53	B. 47	C. 41		D. 3	39	E) NOTA

- 10. Select any prime number greater than 3. Square it and subtract 1. What is the largest number that must be a divisor of the result?
 - A. 1 B. 8 C. 12 D. 24 E) NOTA

 $10\frac{1}{2}$. Are there any perfect squares with an even number of positive integral divisors?

Please fill in this answer for question number four.

11. Question eleven does not exist, however there is an answer for it.

Thank you again for your patience and always remember that you are brilliant. This is the end of the test!

- 12. Which of the following sets contains a palindrome, an ambigram, and a vampire? If you do not know enough information to answer this question, please go on to the next question for the time being.
 - A. {121, 101, 666}C. {1331, 6889, 2187}E. NOTAB. {14641, 80608, 1827}D. {161051, 168891, 2187}
- 13. The square of a number is 8 less than the sum of the first 10 primes. What is that number?
 - A. 11 B. -11 C. \sqrt{109} D. \sqrt{166} E. NOTA

14. What is the maximum number of acute angles that any convex polygon can have?

A. 2 B. 3 C. 4 D. No Limit E. NOTA

During my time in Hawaii, I, Mr. Snube, happened upon a wrinkly old man guarding a bridge. "In order to pass to the ruins beyond," he said "You must answer me these word problems three..."

- 15. "The sum of three four digit numbers is a five digit number," the old man said. "Each of the three four digit numbers has only one distinct digit. Furthermore, the middle three digits of the five digit number are the same as the digits of one of the four digit numbers, and the first and last digits of the five digit number are the same as the digits in the other two four digit numbers, respectively. What is the sum of the three digits used?
 - A. 15 B. 16 C. 17 D. 18 E. NOTA"
- 16. "Three brothers, Aaron, Baron, and Caron are all in the construction business," the old man said. "When the three of them work together, they can build fourteen houses in sixteen days. When the older brother, Caron, works alone, he can build three houses in eight days. Knowing that Aaron and Baron are twins (and thus work at the same rate), tell me how long it would take either Aaron or Baron to build fourteen houses alone.
 - A. 64 days B. 56 days C. $37\frac{1}{3}$ days D. 28 days E. NOTA"
- 17. "There is a five-digit number that can be multiplied by 100,004 to yield a palindrome whose digits are those of the five-digit number followed by those same digits in the reverse order." the old man said. "Which of the following is not a digit of the five-digit number:
 - A. 1 B. 2 C. 5 D. 7 E. 9 ?"

Word problems take me a long time, so I skipped that bridge's questions and came back later. I hope you skipped them as well, because the following are visual helpers for the word problems he gave me:

15. AAAA BBBB +CCCC CBBBA

16			0 days	8 days	16 days						
		A, B, & C	0 houses		14 houses						
		C only	0 houses	3 houses							
		A & B	0 houses								
		A only	0 houses								
17			ABCDE		ABCDE		EDC	BA			
			x 4		x 100000	+ ABC					
		x4x100000+ABCDE00000EDCBAABCDE00000ABCDEEDCBA									
Ple	Please continue with the rest of the test. Thank you for your ever-enduring patience.										
18	18. For how many integers <i>n</i> , such that $2 < n < 100$, does the expression $\sqrt{1 + 2 + + n}$ simplify to an integer?										
	U										
	A. 0		B . 1	C	. 2	D. 3		E)	NOTA	ł	
19	19. A circle contains five chords. What is the maximum number of distinct regions into which these chords can divide the circle?										
	A 1.C		D 10	C	10	D 25		Г	NOTA		
	A. 16		B. 19	C	. 13	D. 25		E.	NOTA	X	
20	20. If the integers a, b, and c are all powers of 2, and $3a + 4b = 5c$, find the least possible value of $a + b + c$.										
	A. 2		B. 5/4	C.	. 5/2	D. 5		E.	NOTA	1	
21.	21. The average of 10 numbers is 63. The average of 6 of these numbers is 57. Find the average of the other 4.										
	A. 66		B. 68	C	. 70	D. 73		E.	NOTA	A	
				-							
22	. Find the s	mallest fracti	on that is a po	ositive integ	gral multiple of	each of the	e fractior	$11^{-1}, \frac{6}{2}$	$\frac{5}{22}$, and	$\frac{4}{33}$.	
But first, set down your pencil and clap your hands three times. If no one has done this yet, then that means you must be the farthest along. Congratulations. Now you can let everyone else know just how smart and fast you are.											
	A. 20/11		B. 30/11	C	. 60/11	D. 90/1	1	E.	NOTA	X	
23	. Find the s	sum of all o	f the positive	e integral f	factors of 120.						
	1 260		P 105	C	226	D 240		Б		L .	
	A. 360		B.195	U.	. 336	D. 240		E.	NOTA	7	

- 24. A positive integer such as 4334 is called a *palindrome* because it reads the same forward and backward. Find the number of digits of the only prime palindrome with an even number of digits.
 - A. 0 B. 2 C. 4 D. 6 E. NOTA
- 25. A positive integer is called *cute* if it has exactly four positive integral divisors. How many of the first 25 positive integers are cute?
 - A. 9 B. 8 C. 7 D. 6 E. NOTA

26. There are eight numbers which can be written with four 2's and no other symbols. These eight numbers are: 2222, 222², 22²², 2^{22²}, 2^{2²}, 2^{2²}

A. 22²² B. 2²²² C. 2²²² D. 2²²² E. NOTA

- 27. Find the smallest positive integer n such that 2n is a perfect square, 3n is a perfect cube, and 5n is a perfect fifth power.
 - A. 2¹⁵3²⁰5²⁴ B. 2⁴⁵3⁵⁰5²⁴ C. 2¹3²5⁴ D. 30³⁰ E. NOTA
- 28. What is the smallest positive integer *n* such that 16!/n is a perfect square?
 - A. 715 B. 1430 C. 5720 D. 7150 E. NOTA
- 29. Consider all the whole numbers from 0 to 1 billion. What is the sum of all the digits needed to write down these numbers?

A. 45,000,000,001 B. 40,500,000,001 C. 45 D. 4,500,000,001 E. NOTA

- 30. When 2^{1990} is multiplied by 5^{1991} , the product has 1991 digits. What is the sum of all these digits?
 - A. 1 B. 8960 C. 5 D. 9955 E. NOTA

(Continue to the next page)

Appendix A:

If you are **bold** enough, you may be able to find a secret word scramble hidden in this test. Unscrambling this secret will give you the answer to question number five. Here is something to help you:

1	2	3	4	5	6	7	8	9	10	11	12	13
А	В	С	D	Е	F	G	Η	Ι	J	Κ	L	Μ
1	1	1	1	1	1	2	2	2	23	24	25	26
Ν	0	Р	Q	R	S	Т	U	V	W	Х	Y	Ζ

Appendix B:

There are certain pairs of two-digit numbers that, when multiplied together produce a four digit number whose digits are the same as those in the original two 2-digit numbers. For example, 21*87=1827. In this example, 1827 is called a vampire, and 21 and 87 are its fangs.

Please return to question number five.