



Mu Alpha Theta
2006 National Convention

Equations and Inequalities
Theta Division
Answers

1. A
2. C
3. B
4. C
5. B
6. E
7. D
8. A
9. B
10. A
11. B
12. A
13. A
14. D
15. E
16. A
17. B
18. E
19. D
20. B
21. C
22. C
23. D
24. B
25. E
26. A
27. C
28. D
29. E
30. C



1. $(4^{\sqrt{2}})^{\sqrt{3}} \rightarrow 4^{\sqrt{6}} \rightarrow 2^{2\sqrt{6}} \rightarrow 2^{\sqrt{24}} \rightarrow 2^{\frac{1}{24^2}} = x \Rightarrow \log_2 x = \frac{1}{24^2}$. (A)
2. The midpoint (h, k) is $(7, 7)$. If the two legs have length 7 and 7, the hypotenuse must be $7\sqrt{2}$. (C)
3. The x -intercepts are determined from the numerator. $2x - 3 = 0 \Rightarrow x = 1.5$. (B)
4. If $\frac{1}{x} + \frac{1}{y} = \frac{7}{12}$, then $\frac{y+x}{xy} = \frac{7}{12}$. Since $xy = 12$, $x+y = 7$. (C)
5. $C_{old} = 2\pi r = 10 \rightarrow r = \frac{5}{\pi}$. $SA_{old} = 4\pi r^2 = 4\pi \left(\frac{5}{\pi}\right)^2 = \frac{100}{\pi}$. $V_{old} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \left(\frac{5}{\pi}\right)^3 = \frac{500}{3\pi^2}$.
 $SA_{new} = \frac{200}{\pi} = 4\pi R^2 \rightarrow R^2 = \frac{50}{\pi^2} \rightarrow R = \frac{5\sqrt{2}}{\pi}$. $V_{new} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \left(\frac{5\sqrt{2}}{\pi}\right)^3 = \frac{1000\sqrt{2}}{3\pi^2}$.

$$\frac{V_{new}}{V_{old}} = \frac{\frac{1000\sqrt{2}}{3\pi^2}}{\frac{500}{3\pi^2}} = 2\sqrt{2}$$
. (B)
6. $|x-4| \geq 2$ and $|x-4| \leq 8$
 $x-4 \geq 2$ or $x-4 \leq -2$ $x-4 \leq 8$ and $x-4 \geq -8$
 $x \geq 6$ or $x \leq 2$ $x \leq 12$ and $x \geq -4$

Solution set: $[-4, 2] \cup [6, 12]$. There are 14 integers in the solution set. (E)

7. If $f(x) = \frac{1}{x}$ and $g(x) = x^2 - 4$, then $f(g(x)) = \frac{1}{(x^2 - 4)}$. (D)
8. $A = \frac{10}{3 + \frac{10}{3 + \frac{10}{3 + \dots}}}$, so $A = \frac{10}{3 + A}$. $3A + A^2 = 10 \rightarrow A^2 + 3A - 10 = 0 \rightarrow (A+5)(A-2) = 0 \Rightarrow A = 2$.

$$B = \sqrt{210 + \sqrt{210 + \sqrt{210 + \dots}}}, \text{ so } B = \sqrt{210 + B}. \quad B^2 = 210 + B \rightarrow (B-15)(B+14) = 0 \Rightarrow B = 15.$$

- $$\frac{C}{D} = 0.\overline{34} = \frac{31}{90}. \quad A = 2, B = 15 = 3 \cdot 5, D = 2 \cdot 3^2 \cdot 5. \text{ LCM} = 2 \cdot 3^2 \cdot 5 = 90. \quad (\text{A})$$
9. $\frac{x+4}{x^3 - 2x^2 + x} = \frac{x+4}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \rightarrow x+4 = A(x-1)^2 + Bx(x-1) + Cx$.
 $x+4 = A(x-1)^2 + Bx(x-1) + Cx = Ax^2 + Bx^2 - 2Ax - Bx + Cx + A$.
 $0 = A+B, 1 = -2A-B+C, 4 = A \Rightarrow (4, -4, 5)$. (B)
 10. The matrix is singular if its determinant is 0. $1(2x-4) - 0 + 2(x-3x) = 0 \rightarrow 2x-4 + 2x-6x = 0$.
 $-2x = 4 \Rightarrow x = -2$. (A)



11. According to the Triangle Inequality Theorem, the sum of the two shortest sides must be greater than the longest side. Since the actual side lengths are unknown, test all three possibilities.

$$(3x+4) + (6x-1) > 8x+2 \rightarrow x > -1$$

$$(6x-1) + (8x+2) > 3x+4 \rightarrow x > \frac{3}{11} \quad \text{The only solution creating viable side lengths is (B).}$$

$$(3x+4) + (8x+2) > 6x-1 \rightarrow x > -\frac{7}{5}$$

12. The center is at $(0, 3)$, so $c = 2$. The length of the major axis is 8. Since $2a = 8$, $a = 4$. Using

$$a^2 = b^2 + c^2, b^2 = 12. \frac{x^2}{16} + \frac{(y-3)^2}{12} = 1 \rightarrow 3x^2 + 4(y-3)^2 = 48 \rightarrow 3x^2 + 4y^2 - 24y - 12 = 0.$$

Multiplying by 4, (A) is obtained.

13. The three lines create a right triangular region with vertices $(-2, 0)$, $(0, 0)$ and $(0, -2)$. Each leg has length 2, so the area is $\frac{1}{2}(2)(2) = 2$. (A)

$$14. F = \frac{kV^2}{R} \rightarrow 600 = \frac{15^2 k}{3} \rightarrow k = 8 \Rightarrow F = \frac{(8)25^2}{4} = 1250. \quad (\mathbf{D})$$

$$15. (15+5)T_1 = 35 \rightarrow T_1 = \frac{7}{4} \text{ hr}. \quad (15-5)T_2 = 35 \rightarrow T_2 = \frac{7}{2} \text{ hr}. \quad R = \frac{D}{T} = \frac{\frac{70}{4}}{\frac{7}{4} + \frac{7}{2}} = \frac{\frac{70}{4}}{\frac{21}{4}} = \frac{280}{21} = 13.\bar{3}. \quad (\mathbf{E})$$

16. $\sqrt{4x+7} \leq 4 \rightarrow 4x+7 \leq 16 \rightarrow x \leq 2.25$. Testing the domain, $4x+7 \geq 0 \rightarrow x \geq -1.75$. The integers are -1 , 0 , 1 , and 2 , the average being 0.5 . (A)

$$17. -4x^2 + 17x + 15 \leq 0 \rightarrow 4x^2 - 17x - 15 \geq 0 \rightarrow (4x+3)(x-5) \geq 0 \Rightarrow \left(-\infty, -\frac{3}{4}\right] \cup [5, \infty) \cdot \left[-\frac{3}{4}\right] = -1. \quad (\mathbf{B})$$

$$18. (x^2 - 6x + 4)^2 = 16 \rightarrow x^2 - 6x + 4 = \pm 4. \quad \text{If } +4, \text{ then } x^2 - 6x = 0 \rightarrow x = 0, 6. \quad \text{If } -4, \text{ then } x^2 - 6x + 8 = 0 \rightarrow (x-4)(x-2) = 0 \rightarrow x = 2, 4. \quad 0 + 6 + 2 + 4 = 12. \quad (\mathbf{E})$$

$$19. \frac{x^2 - 4}{x^2 - 1} \leq \frac{x}{x+3} \rightarrow \frac{3(x^2 - x - 4)}{(x+1)(x-1)(x+3)} \leq 0. \quad \text{The critical values are } x = -3, \frac{1 \pm \sqrt{17}}{2}, -1, 1. \quad \text{Using } \sqrt{17} \approx 4.1 \text{ and performing a +/- analysis, it is found that the most positive value of } x \text{ is } \frac{1 + \sqrt{17}}{2}. \quad (\mathbf{D})$$



20. $2^{x+3} = 3^{2x-1} \rightarrow 2^x 2^3 = 3^{2x} 3^{-1} \rightarrow 24 = \frac{3^{2x}}{2^x} = \frac{9^x}{2^x} = \left(\frac{9}{2}\right)^x \rightarrow \log 24 = x \log\left(\frac{9}{2}\right) \Rightarrow x = \frac{\log 24}{\log\left(\frac{9}{2}\right)}$. (B)

21. $3^x 4^y = 15,552 = 243 \cdot 64 = 3^5 \cdot 4^3 \Rightarrow 5 + 3 = 8$ (C)

22. $3^{5x} \cdot 81^{-x} \geq 9^{x^2-3} \rightarrow 3^{5x} 3^{-4x} \geq 3^{2x^2-6} \rightarrow x \geq 2x^2 - 6 \rightarrow 2x^2 - x - 6 \leq 0 \rightarrow (2x+3)(x-2) \leq 0$.

$\left[-\frac{3}{2}, 2\right] \rightarrow -1, 0, 1, 2$. Positive integers are 0, 1, 2, so (C).

23. $w = 17 + 2(3) + \left\lceil \frac{3(3+1)}{5} \right\rceil + 1973 + \left\lceil \frac{1973}{4} \right\rceil - \left\lceil \frac{1973}{100} \right\rceil + \left\lceil \frac{1973}{400} \right\rceil + 2$
 $\rightarrow 17 + 6 + [2.4] + 1973 + [493.25] - [19.73] + [4.9325] + 2 = 23 + 2 + 1973 + 493 - 19 + 4 + 2 = 354$.
 $2473 \div 7 = 354R0$, so Saturday (D).

$$\begin{array}{r} 4 \\ \hline 1 & -11 & 30 \end{array}$$

24. Using synthetic division,
$$\begin{array}{r} 4 - 28 \\ \hline 1 & -7 & 2 \end{array} \Rightarrow y = x - 7$$
 (B)

25. Using the factors of 32 (p) and the factors of 16 (q), $\frac{p}{q}$ could be any of the choices given, so (E).

26. The point $(0, 7)$ is on the line $y = \frac{1}{2}x + 7$. Find the equation of a line that passes through $(0, 7)$ and is perpendicular to $y = \frac{1}{2}x + 7$. That line is $y = -2x + 7$. The lines $y = -2x + 7$ and $2y = x + 6$ intersect at $\left(\frac{8}{5}, \frac{19}{5}\right)$. The distance between $\left(\frac{8}{5}, \frac{19}{5}\right)$ and $(0, 7)$ is $\frac{8\sqrt{5}}{5}$. (A)

27. $3y + 9 = -2x \rightarrow 3y = -2x - 9 \rightarrow y = -\frac{2}{3}x - 3$. $m = -\frac{2}{3}$, so $m_{\perp} = \frac{3}{2}$. (C)

28. $f(x+1) - f(x) = |3(x+1)+1| - 5 - (|3x+1|-5) = |3x+4| - |3x+1|$. (D)

29. $f(x) = y = \sqrt{2x-1} \Rightarrow x = \sqrt{2y-1} \rightarrow x^2 = 2y-1 \rightarrow y = \frac{x^2+1}{2}$. (A)

30. Let k be the proportionality constant. If $4k = c$, $bk = -7$, and $ck = 16$, then $k = \frac{c}{4}$, $k = -\frac{7}{b}$, and $k = \frac{16}{c}$. Now, $\frac{c}{4} = \frac{16}{c}$, so $c = \pm 8$. If $c = 8$, $k = 2$ and $b = -\frac{7}{2}$; if $c = -8$, $k = -2$ and $b = \frac{7}{2}$. There are two values for b and c , so (C).