



Mu Alpha Theta
2006 National Convention

Functions
Theta Division

1. For $f(x) = \frac{x-1}{3}$, for what value of x is $f(x) = 30$?
- A. $\frac{29}{3}$ B. $\frac{31}{3}$ C. 89 D. 91 E. NOTA
2. For $g(x) = \sqrt{x+1} - 1$ find $g(g(3))$.
- A. 2 B. $\sqrt{2} - 1$ C. 1 D. 0 E. NOTA
3. $f(x) = \begin{cases} x-2 & \text{for } x > 1 \\ 4-x & \text{for } x \leq 1 \end{cases}$. If for $a \neq b$, $f(a) = f(b) = 3$ then give the value of $|a-b|$.
- A. 0 B. 1 C. 4 D. 6 E. NOTA
4. The function $A(x)$ gives the area of a triangle with vertices on the coordinate axes at $(0,0)$, $(0, x)$ and $(x,0)$, for $x > 0$. For what value of x is A equal to 20 ?
- A. $\sqrt{10}$ B. $2\sqrt{10}$ C. 20 D. 40 E. NOTA
5. For $f(x) = x^2 - 2x + 3$ which is true for all values of x ?
- A. $f(-x) = -f(x)$ B. $f(x) = f(-x)$ C. $f(1+x) = f(1-x)$ D. $f(x) \geq 3$ E. NOTA
6. For $f(x) = 3^{x+1}$, give the value of $\frac{3f(3)}{f(2)}$.
- A. 9 B. 6 C. 3 D. 1 E. NOTA
7. The domain of $f(g(x))$ contains all reals except for x-values a and b .
For $f(x) = \frac{1}{x-3}$ and $g(x) = \frac{x+3}{2x+2}$ give the value of ab .
- A. 0.5 B. 0.6 C. -0.6 D. -1.8 E. NOTA
8. Relations g , h , and k have the properties below. Which cannot be a function?
- $g(2) = g(-2)$ $h(x) = \pm\sqrt{x}$ $k(-3) = -k(-3)$
- A. g only B. h only C. k only D. h and k only E. NOTA



9. $f(x) = \log x$ and $f(k) = \log 1 + \log 2 + \log 3 + \dots + \log n$. If $k = 120$ then give the value of n .

- A. 4 B. 5 C. 59 D. 119 E. NOTA

10. $A(x)$ is given by the surface area of a sphere of radius x , divided by the radius of the sphere, for domain $x > 0$. For what value of x is $A(x)$ equal to the area of the great circle of the sphere?

- A. $\sqrt{4\pi}$ B. 1 C. 2 D. 4 E. NOTA

11. $f(x) = 1 - x^2$ and $g(x) = f(x) + 2x$. Find the value of x such that $f(x) = g(x) - 2$.

- A. 1 B. 2 C. 3 D. 4 E. NOTA

12.

x	1	2	3	4
f	4	6	1	5
g	a	b	c	d

In the table above, relations f and g have the property that $f(g(x)) = g(f(x)) = x$, for all x . Give the value of $a + d$.

- A. 10 B. 5 C. 4 D. 3 E. NOTA

13. For $f(x) = x \cdot 16^x$ find $f(f(\frac{1}{4}))$.

- A. $\sqrt[4]{2}$ B. $\sqrt{2}$ C. 2 D. 4 E. NOTA

14. f is a linear function with domain all reals, and $f(1) = 2$. If $f(-x) = -f(x)$ for all numbers in its domain, find the value of k where $f(k) = 12$.

- A. 6 B. 11 C. 12 D. 24 E. NOTA

15. The function f is defined as the value of $C(x, 3)$, the number of different combinations of x objects taken 3 at a time, $x > 3$. Find the greatest value of k such that $f(k) < 10$.

- A. 3 B. 4 C. 5 D. 6 E. NOTA

16. Given $f(a,b) = a^2 - 2b + 1$. If $f(k,1) = 12$ then what is the value of $k^2 - 9$?

- A. 3 B. 3 or -3 C. 4 D. 13 E. NOTA



17. The function $f(x)$ is defined as the length of side \overline{AC} of triangle ABC, if $AB=10$, $BC=10$ and $m\angle B = x$, for domain $0 < x < 180$ degrees. Find $f(60)$.

- A. 10 B. 50 C. $\frac{20\sqrt{2}}{3}$ D. $25\sqrt{3}$ E. NOTA

18. What is the maximum value of the function $f(x) = 9 - 2x - x^2$?

- A. 11 B. 10 C. 9 D. 8 E. NOTA

19. Which function $y = f(x)$ below has an inverse that is not a function?

- A. $y = \frac{1}{x}$ B. $y = \frac{1}{2}x - 1$ C. $y = x^2$ D. $xy - 2 = 0$ E. NOTA

20. For $f(x) = ax^3 + 4x^2 + bx + c$, if $f(1) = 6$ then give the value of $a + b + c$.

- A. 8 B. 6 C. 4 D. 2 E. NOTA

21. $g(x) = f(x) + 1$ and f is a linear function with y-intercept 0 and positive slope. If the area bounded by f , the x -axis, and the line $x = 2$ is k , then find the area bounded by the graph of g , the axes, and the line $x = 2$.

- A. $k + 1$ B. $k + 2$ C. $k + 3$ D. $k + 4$ E. NOTA

22. $f(x) = a|x+b|+c$ and has minimum value at the point $(1, -4)$ and an x-intercept 2, then $f(3) = ?$

- A. 0 B. 1 C. 4 D. 12 E. NOTA

23. $P(x)$ gives the percent of decrease when 400 is decreased to x , for $0 < x < 400$. $P(300) = ?$

- A. 25 B. 30 C. 50 D. 100 E. NOTA

24. $f(x) = \sum_{n=0}^x \left(\frac{1}{3}\right)^n$. Which value is closest to $f(10^{100})$?

- A. 0 B. 1.5 C. 3 D. 6 E. NOTA



25. $f(x) = x + (x-1)i$ and $i = \sqrt{-1}$. Find $\left| \frac{f(2)}{f(0)} \right|$.
- A. $2+i$ B. $1+2i$ C. $\sqrt{5}$ D. 5 E. NOTA
26. The function $f(x)$ is defined as the length of side \overline{AC} of triangle ABC, if $AB=12$, $BC=10$ and $m\angle B = x$, for domain $0 < x < 180$ degrees. Give the range of the function f , in interval notation.
- A. $(2, 22)$ B. $(0, 12)$ C. $(0, 22)$ D. $(2, 12)$ E. NOTA
27. A parabolic arch (vertex at maximum height) has vertex 200 feet off of the ground. Let $H(x)$ be the height in feet of the arch above ground at x feet from the center (axis) of the arch. If $H(200) = 0$ then find the value of $H(10)$
- A. 200 B. 199.5 C. 195 D. 105 E. NOTA
28. For $x > 0$, $f(x) = \sqrt{x - \sqrt{x - \sqrt{x - \sqrt{\dots}}}}$ If $f(k) = 2$ then $k =$
- A. 1 B. $\sqrt{2}$ C. 3 D. 6 E. NOTA
29. $f(x, y) = 27^{\frac{x}{y}}$ and $g(x, y) = 3^{\frac{y}{x}}$. If $f(k, 3) = g(2, k) \cdot \sqrt{3}$, then $g(k, 2) =$
- A. $\frac{1}{9}$ B. $\sqrt{3}$ C. 3 D. 9 E. NOTA
30. If $f(x+1) = \frac{1}{x} + \frac{1}{x+1}$ and the domain of $f(x)$ is positive integers where f is defined, which could NOT be a value of the range of $f(x)$?
- A. $\frac{19}{90}$ B. $\frac{17}{72}$ C. $\frac{7}{12}$ D. $\frac{13}{30}$ E. NOTA