



- 1) C
- 2) A
- 3) D
- 4) A
- 5) C
- 6) B
- 7) B
- 8) B
- 9) A
- 10) C
- 11) C
- 12) C
- 13) C
- 14) D
- 15) B
- 16) C
- 17) B
- 18) D
- 19) A
- 20) C
- 21) C
- 22) A
- 23) A
- 24) A
- 25) B
- 26) E
- 27) A
- 28) B
- 29) C
- 30) B



1)  $(1 + i)^2 = 2i$      $(2i)^{10} = -1024$     **C**

2) Fermat's Last Theorem : There are no non-zero integer solutions to  $x^n + y^n = z^n$  when  $n > 2$ .    **A**

3)  $P(\text{at least 2 were born on the same day}) = 1 - P(\text{all born on different days}) = 1 - \left(1 \cdot \frac{6}{7} \cdot \frac{5}{7} \cdot \frac{4}{7}\right) = \frac{223}{343}$     **D**

4)  $ax^2 + cx^2 + 10x^2 = 8x^2$

$$\begin{array}{rcl} ax &+& bx - 2x = -3x \\ a &+& b + 2c = -11 \end{array}$$

$$\begin{array}{rcl} a &+& c + 10 = 8 \\ a &+& b - 2 = -3 \end{array}$$

$$\begin{array}{rcl} a &+& b + 2c = -11 \\ a &+& c = -2 \end{array}$$

$$a + b = -1$$

$$a + b + 2c = -11$$

$$\Rightarrow a = 3, b = -4, c = -5$$

$$\begin{aligned} \rightarrow 4(3) + (-4) - (-5) &= 13 & \mathbf{A} \end{aligned}$$

5)  $V_{\text{cylinder}} = \pi r^2 h = \pi(9)(10) = 90\pi$      $V_{\text{cone}} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi(9)(5) = 15\pi$      $90\pi - 30\pi = 60\pi$     **C**

6)  $25x^2 - 150x + 4y^2 + 32y + 189 = 0$      $\rightarrow \left(\frac{x-3}{2}\right)^2 + \left(\frac{y+4}{5}\right)^2 = 1$      $A = r_x r_y \pi = (2)(5)\pi = 10\pi$     **B**

7)  $(-1)^{2006} + 1 = 2$     **B**

8)  $2^6 - 1 = 63$ . Proper subsets do not include the set itself.    **B**

9)  $25x^2 + 9y^2 - 200x + 18y + 184 = 0 \rightarrow \left(\frac{x-4}{3}\right)^2 + \left(\frac{y+1}{5}\right)^2 = 1$  center : (4, -1)

$$9x^2 - 4y^2 + 90x + 32y + 197 = 0 \rightarrow -\left(\frac{x+5}{2}\right)^2 + \left(\frac{y-4}{3}\right)^2 = 1$$
 center : (-5, 4)

$$m = \frac{-1 - 4}{4 - (-5)} = \frac{-5}{9}; \quad y + 1 = \frac{-5}{9}(x - 4) \rightarrow 5x + 9y = 11 \quad \mathbf{A}$$

10)  $V_{\text{block}} = (30 \cdot 20 \cdot 20) - (2 \cdot 10 \cdot 10 \cdot 20) = 8000 \text{ cm}^3$  Since  $1 \text{ m}^3 = 1,000,000 \text{ cm}^3$ ,  $8000 \text{ cm}^3 = 0.008 \text{ m}^3$   
 $(0.008)(1700) = 13.6 \text{ kg.}$     **C**

11) **BC**    **Bc**    **bC**    **bc**  
**BC**    BBCC    BBCc    BbCC    BbBc  
**Bc**    BBCc    BBcc    BbCc    Bbcc

**bC**    BbCc    BbCc    **bbCC**    **bbCc**     $P(\text{yellow feathers}) = \frac{3}{16}$     **C**

**bc**    BbCc    Bbcc    **bbCc**    bbcc



Mu Alpha Theta  
2006 National Convention

Solutions  
Gemini  
Theta Division

12)  $342_7 = (3 \cdot 7^2) + (4 \cdot 7) + 2 = 177$

$177 \div 128 = 1$  (remainder 49)

$49 \div 64 = 0$  (remainder 49)

$49 \div 32 = 1$  (remainder 17)

$17 \div 16 = 1$  (remainder 1)

$1 \div 8 = 0$  (remainder 1)

$1 \div 4 = 0$  (remainder 1)

$1 \div 2 = 0$  (remainder 1)

$1 \div 1 = 1$  (remainder 0)

$\Rightarrow 10110001 \quad \mathbf{C}$

13)  $69 + 50 + 55 - 26 - 15 - 27 + 11 + 33 = 150 \quad \mathbf{C}$

14)  $y = 3(x^2 + 8x) + 50 = 3(x^2 + 8x + 16) + 50 - 48 = 3(x + 4)^2 + 2$   
vertex is at (-4, 2)

distance from vertex to focus is  $\frac{1}{4(3)} = \frac{1}{12}$  parabola turns up, so focus is at  $(-4, 2\frac{1}{12}) \quad \mathbf{D}$

15) Won's journey forms a right triangle, 9 X 40.  $9^2 + 40^2 = 1681 \Rightarrow 41 \quad \mathbf{B}$

16)

$$103\frac{3}{7} = 81\frac{2}{7} + (6 - 1)d$$

$$4\frac{3}{7} = d$$

$$a = 85\frac{5}{7}, b = 90\frac{1}{7}, c = 94\frac{4}{7}, d = 99$$

$$\text{sum is } 369\frac{3}{7} \quad \mathbf{C}$$

17)  ${}_7C_4 (x^4)^3 \left(\frac{1}{x^3}\right)^4 = \frac{7!}{4!3!} = 35 \quad \mathbf{B}$

18) X **D**

19)

$$\begin{aligned} & \frac{10}{\sqrt[3]{2} - \sqrt[3]{7}} \cdot \frac{\sqrt[3]{4} + \sqrt[3]{14} + \sqrt[3]{49}}{\sqrt[3]{4} + \sqrt[3]{14} + \sqrt[3]{49}} \\ & \frac{10(\sqrt[3]{4} + \sqrt[3]{14} + \sqrt[3]{49})}{-5} = -2\sqrt[3]{4} - 2\sqrt[3]{14} - 2\sqrt[3]{49} \quad \mathbf{A} \end{aligned}$$

20)  $p^2 - 2 = p \quad p^2 - p - 2 = 0 \quad (p - 2)(p + 1) = 0 \quad p = 2, -1 \leftarrow \text{extraneous} \quad \mathbf{C}$



21)  $A = \frac{180(8-2)}{8} = 135$        $D = \frac{8(8-3)}{2} = 20 \rightarrow 135 + 20 = 155$       **C**

22) determinant[A] =  $12 - 6 = 6$

$$\frac{1}{6} \begin{bmatrix} 3 & 2 \\ 3 & 4 \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ 2 & 3 \\ \frac{1}{2} & \frac{2}{3} \\ 2 & 3 \end{bmatrix} \quad \mathbf{A}$$

23)  $2^{11} = 2048$       **A**

24)  $f(x) = \frac{(x+1)(x-1)}{(x-4)(x-1)}$       hole:  $\left(1, \frac{-2}{3}\right)$ ; asymptote:  $x = 4 - 1 + \frac{-2}{3} + 4 = 4\frac{1}{3}$       **A**

25 Draw  $\overline{CA}, \overline{ND}, \overline{NB}$ ;  $CA=r$ ,  $ND=r-4$ ,  $NB=x$ ,  $PB=12+x$ ,  $AN=r+4$

$$(PT)^2 = (PB)(PJ) \rightarrow 12^2 = (12+x)(8) \rightarrow x = 6 = NB \quad (NJ)(NB) = (AN)(ND) \rightarrow (4)(6) = (r+4)(r-4)$$

$$\rightarrow r = \sqrt{40} = 2\sqrt{10} \quad \mathbf{B}$$

26)

$$\frac{\frac{u+v}{u-v} - \frac{u-v}{u+v}}{\frac{u+v}{u-v} + \frac{u-v}{u+v}} \cdot \frac{(u-v)(u+v)}{(u-v)(u+v)} =$$

$$\frac{u^2 + 2uv + v^2 - (u^2 - 2uv + v^2)}{u^2 + 2uv + v^2 + (u^2 - 2uv + v^2)} =$$

$$\frac{4uv}{2(u^2 + v^2)} = \frac{2uv}{u^2 + v^2} \quad \mathbf{E}$$

27)  $(0.10)(x) + (1)(6-x) = (.40)(6) \quad 0.1x + 6 - x = 2.4 \quad x = 4 \quad \mathbf{A}$

28)  $r = \frac{kp}{q} - 27 = \frac{k(3)}{4} - 36 = k$

$$r = \frac{(36)(2)}{9} = 8 \quad \mathbf{B}$$

29) 42      **C**

30) Let  $QR=x$  and  $PQ=21-x$        $h^2 + x^2 = 169$        $h^2 + (21-x)^2 = 400$        $h^2 + 441 - 42x + x^2 = 400$   
 $169 + 441 - 42x = 400 \quad x = 5 \rightarrow h=12 \quad \mathbf{B}$