



**Mu Alpha Theta
2006 National Convention**

**Logs and Exponents
Theta Division
Answers**

1. B
2. D
3. D
4. C
5. C
6. B
7. D
8. A
9. B
10. C
11. A
12. C
13. C
14. C
15. B
16. B
17. D
18. B
19. C
20. A
21. D
22. C
23. A
24. B
25. A
26. A
27. E
28. D
29. B
30. A



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Solutions

1. $5^{x+2} - 5^x < 5$ $5^x (5^2 - 1) < 5$ $5^x (24) < 5$ $5^x < \frac{5}{24}$ The largest x is -1.

2. $A = \left(\frac{1}{2} - \frac{1}{3}\right)^{-1} = \left(\frac{1}{6}\right)^{-1} = 6$ $B = \left(\frac{5}{6}\right)^{-1} = \frac{6}{5}$ $C = [3(5\sqrt{5}) - 2(3\sqrt{5}) - 4\sqrt{5}]^2 = [15\sqrt{5} - 6\sqrt{5} - 4\sqrt{5}]^2$

$$= [5\sqrt{5}]^2 = 125 \quad D = (3\sqrt{5} + 3)(3\sqrt{5} - 3) = 45 - 9 = 36 \quad \frac{ABC}{D} = \frac{6\left(\frac{6}{5}\right)(125)}{(36)} = 25$$

3. $\left[\frac{x^{3k-2}}{x^{2k+1}} \right] \left[\frac{x^{k-3}}{x^{2k+2}} \right] = \frac{x^{4k-5}}{x^{4k+3}} = x^{-8} = x^y \quad \therefore y = -8$

4. $\log 10 + \log A - \log B + 3 \log A - \log 100 - \log C = 1 + 2.5 - 1.2 + 7.5 - 2 - 3.6 = 4.2 \quad \text{or } \frac{21}{5}$

5. Let $x = \sqrt[3]{y}$, then $y = x^3$ and $(\sqrt[3]{y})^y = 3$, cubing $y^y = 3^3$, or $y = 3$ and $x = \sqrt[3]{3}$

6. $2^x = 1 * 2^3 = 8$, $5^y = 1 * 5^{-2} = \frac{1}{25}$, $2^x * 5^y = \left\{ \frac{8}{25} \right\}$

7. $(x+1) = (x-1)^2$, $x+1 = x^2 - 2x + 1$, $x^2 - 3x = 0$, $x = 3, 0$ only $x = 3$

8. $\log_8 144 - \log_8 9 = \log_8 \frac{144}{9} = \log_8 16 = \frac{\log 16}{\log 8} = \frac{4 \log 2}{3 \log 2} = \frac{4}{3}$

9. $(1+t)^x = \frac{b}{a}$ $x \log(1+t) = \log\left(\frac{b}{a}\right)$ $x = \frac{\log\left(\frac{b}{a}\right)}{\log(1+t)}$

10. $\log(0.75) = \log \frac{3}{2^2} = \log 3 - 2 \log 2 = 0.477 - 0.602 = -0.125$

11. $2^x = 2^{2y+2}$, $x = 2y + 2$, $x = (y+1)^2$, $x = y^2 + 2y + 1$,

$2y + 2 = y^2 + 2y + 1$ $y^2 = 1$, $y = \pm 1$, if $y = -1$, $x = 0$, if $y = 1$, $x = 4$, the sum is 4

12. $M(4) \rightarrow 4 = 2^2 \rightarrow \log 2$; $M(6) \rightarrow 6 = 3^2 \rightarrow \log 3$

$\log 2 + 0 + \log 3 = \log 6$

13. $\log \frac{50}{7} = \log \frac{2(5)^2}{7} = \log 2 + 2 \log 5 - \log 7 = a + 2b - c$

14. $\frac{9^{x+2}}{(3^{x-1})^x} \div \frac{(27^x)^{x+1}}{81^{x^2-1}} = \frac{3^{2x+4}}{3^{x^2-x}} * \frac{3^{4x^2-4}}{3^{3x^2+3x}} = \frac{3^{4x^2+2x}}{3^{4x^2+2x}} = 1$

15. $\log_3 (9\sqrt{27})(\sqrt[3]{3}) = \log_3 \left(3^2 \cdot 3^{\frac{3}{2}} \cdot 3^{\frac{1}{3}} \right) = \log_3 3^{\frac{23}{6}} = \frac{23}{6}$

16. $\left[27^{\frac{2}{3}} + 64^{\frac{2}{3}} \right]^{\frac{3}{2}} - 10^2 = [9 + 16]^{\frac{3}{2}} - 100 = 25^{\frac{3}{2}} - 100 = 125 - 100 = 25$



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17. $8^{3x+2} = 4^{5x-1} \rightarrow 2^{9x+6} = 2^{10x-2} \quad 9x + 6 = 10x - 2 \quad x = 8$

18. $3^x + 3^{-x} = \frac{10}{3} \rightarrow 3^{2x} + 1 - \frac{10}{3}(3^x) = 0 \quad \text{If } y = 3^x \text{ then } y^2 - \frac{10}{3}y + 1 = 0 \rightarrow (3y-1)(y-3) = 0 \Rightarrow y = 1/3 \text{ and } y = 3 \quad \frac{1}{3} = 3^x \quad x = -1, 3^x = 3 \quad x = 1 \quad \text{sum} = 0$

19. $6x^{\frac{-1}{2}} - 17x^{\frac{-1}{4}} = -5 \quad 6x^{\frac{-1}{2}} - 17x^{\frac{-1}{4}} + 5 = 0 \quad \left(3x^{\frac{-1}{4}} - 1\right)\left(2x^{\frac{-1}{4}} - 5\right) = 0$

$$x^{\frac{-1}{4}} = \frac{1}{3} \quad x = 81; \quad x^{\frac{-1}{4}} = \frac{5}{2} \quad x = \frac{16}{625}$$

20. $x = \left(\frac{1}{2}\right)^4 = \frac{1}{16}, \quad \log_4 \frac{1}{16} = -2$

21. $2 \ln x = (\ln x)^2 \quad (\ln x)^2 - 2 \ln x = 0 \quad \ln x(\ln x - 2) = 0$
 $\ln x = 0 \quad x = 1 \quad \text{or} \quad \ln x = 2, \quad x = e^2$

22. $A = \frac{\log 225}{\log 8} = \frac{2 \log 15}{3 \log 2} = \frac{2}{3}(\log_2 15) = \frac{2}{3}B, \quad \frac{A}{B} = \frac{2}{3}$

23. $\frac{e^x + \frac{1}{e^x}}{e^x - \frac{1}{e^x}} = \frac{\frac{e^{2x} + 1}{e^x}}{\frac{e^{2x} - 1}{e^x}} = \frac{e^{2x} + 1}{e^{2x} - 1} = 2 \quad e^{2x} + 1 = 2e^{2x} - 2 \quad e^{2x} = 3 \quad 2x = \ln 3 \quad x = \frac{1}{2} \ln 3$

24. $3^{20} > (32)^x \quad 3^{20} > 2^{5x} \quad \sqrt[5]{3^{20}} > \sqrt[5]{2^{5x}} \quad 3^4 > 2^x \quad 81 > 2^x \quad 2^6 = 64 \quad 2^7 = 128 \quad \therefore x = 6$

25. $(\ln x)^3 - 4 \ln x = 0 \quad \ln x((\ln x)^2 - 4) = 0 \quad \ln x = 0 \quad x = 1 \quad \text{or} \quad \ln x = \pm 2 \quad x = e^2, \frac{1}{e^2}$

26. $\log_b 14 - \log_b 3 - \log_b b = \log_b 2 + \log_b 7 - \log_b 3 - 1 = 0.2789 + 0.7831 - 0.4421 - 1 = -0.3801$

27. $\ln(x-2) - \ln(e) = \ln(x+2), \quad \ln\left(\frac{x-2}{e}\right) = \ln(x+2), \quad x-2 = e(x+2), \quad x(1-e) = 2+2e,$

$$x = \frac{2+2e}{1-e}$$

28. $y = 9 - 4x \quad 4 + 2y = 3x \quad 4 + 18 - 8x = 3x \quad 22 = 11x \quad x = 2 \quad y = 1$

29. $8e^{2x} + 27 = 35e^x \quad 8e^{2x} - 35e^x + 27 = 0 \quad (8e^x - 27)(e^x - 1) = 0 \quad e^x = \frac{27}{8}$

$$x = \ln\left(\frac{27}{8}\right), \quad e^x = 1 \quad x = 0 \quad \text{sum} = \ln\left(\frac{27}{8}\right)$$

30. $\log_x 3 + \log_x 4 + \log_x 5 = \log_x 60 = \frac{1}{\log_{60} x}$