1. C
2. A
3. C
4. D
5. C
6. B
7. B
8. E ($4/45$)
9. D
10. C
11. B
12. A
13. C
14. B
15. D
16. C
17. B
18. C
19. B
20. E ($4/9$)
21. B
22. C
23. B
24. A
25. B
26. D
27. A
28. C
29. C
30. D

1. **C** The two possibilities are BBB and BBG, the 3rd child is a girl half the time.
2. **A** There are 6! = 720 arrangements of all the digits. 5! = 120 start with a 3, and 5! = 120 end with a 4, but 4! = 24 of these were counted twice. Those elements which were counted twice we don't wish to count at all, so we subtract 24 twice, and find that 120 + 120 – 24 – 24 = 192 combinations meet the desired requirements, so the probability is $4/15$.
3. **C** 1210! ends with $\left\lfloor 1210/5^1 \right\rfloor + \left\lfloor 1210/5^2 \right\rfloor + \ldots = 242 + 48 + 9 + 1 = 300$ zeros, but 1209! ends with 299 zeros, so $n!$ ends with at least 300 zeros iff $n \geq 1210$. There are 791 natural numbers in $[1210, 2001)$, and 2000 natural numbers in $[1, 2001)$, so the probability is $791/2000$.
4. **D** The sum must be 2, 3, 5, 7, or 11, which have respective probabilities: $1/36$, $2/36$, $4/36$, $6/36$, and $2/36$. The sum is $15/36 = 5/12$.
5. **C** A number is divisible by 3 iff the sum of its digits is divisible by 3. These 6 digits add to 21, so a subset of 4 digits will be a multiple of 3 iff the 2 digits not chosen are a multiple of 3. There are ${}_6C_2 = 15$ ways to choose these 2 digits, and the only ways to have the sum a multiple of 3 are: 1, 2; 1, 5; 2, 4; 3, 6; and 4, 5. So the probability is $5/15 = 1/3$.
6. **B** The area within 3 units is 9 square units, the area within 4 is 16. $(16-9)/16 = 7/16$.
7. **B** Regardless of the first card I choose, there are 3 remaining cards of the same rank, of the remaining 51 total choices. $3/51 = 1/17$.
8. **E** ($4/45$) This is only possible if Saahil is in one of the 8 leftmost seats, which will occur with probability $8/10$. If Saahil is in one of these seats, the probability that Saajan is in the seat 2 to his right is $1/9$. So $\left(8/10\right)\left(1/9\right) = 4/45$ is the probability of Saajan being 2 to Saahil's right.
9. **D** $1 - \left(1/2\right)^3 = 7/8$.
10. **C** We're looking for the probability that the first 4 balls are not blue: $7/12\left(6/11\right)\left(5/10\right)\left(4/9\right) = 7/99$.
11. **B** There are 36 choices for each of the first 3 characters and 10 for each of the last 4.
12. **A** Consider placing students around the table one at a time, and find the probability that placing each student preserves the girl-boy ordering. For seat #1, any of the 10 students will preserve the order. For seat #2 (seats ordered in order), we must place a student of opposite gender to seat #1, there are 5 such students unseated of our 9 choices, so the probability of preserving the order at this seat is $5/9$. #3 must be different from #2, which occurs with probability $4/8$, etc. The probability that the order is preserved in all seats is $\left(10/10\right)\left(5/9\right)\left(4/8\right)\left(4/7\right)\left(3/6\right)\left(3/5\right)\left(2/4\right)\left(2/3\right)\left(1/2\right)\left(1/1\right) = 1/126$.
13. **C** If my speed is at least 25 mph I'll travel 200 miles in a day, otherwise my expected distance is 100 miles. So my overall expected distance is .5(200) + .5(100) = 150. $\left\lceil 2500/150 \right\rceil = 17$.

14. **B** $\frac{5(4)+8(7)}{13(12)}=\frac{20+56}{156}=\frac{76}{156}=\frac{19}{39}$

15. **D** First note that any number relatively prime to 2 is relatively prime to 4 (and vice versa) so we count the number of natural numbers less than 2006 that are relatively prime to 2 and 3. By the inclusion-exclusion principal this count is:
$2005-\left\lfloor\frac{2005}{2}\right\rfloor-\left\lfloor\frac{2005}{3}\right\rfloor+\left\lfloor\frac{2005}{6}\right\rfloor=2005-1002-668+334=669$. So the probability is $\frac{669}{2005}$.

16. **C** Here are the 3 cases, each with probability $1/3$:

	#1	#2	#3
A:	$0	$0	$30
B:	$0	$30	$0
C:	$30	$0	$0

Without loss of generality, assume that you initially picked door #1. In case 1 and 2, the host reveals doors #2 and #3 respectively, and in case #1 he reveals either other door. The cases are now:

	#1	other remaining door
A:	$0	$30
B:	$0	$30
C:	$30	$0

If you don't switch, you get $0 $2/3$ of the time and $30 $1/3$ of the time, so 16 is $10. If you do switch, you get $30 $2/3$ and $0 $1/3$ of the time, so 17 is (counter intuitively) $20.

17. **B** There are a total of 3 heads, each of which is equally likely to be showing. Two of these heads belong to the two-headed coin and one does not. Therefore the probability that the coin is the two-headed one is $2/3$.

18. **C** The student has a $1/5$ chance of answering each question correctly, regardless of what the correct answer choice is.

19. **B** There are 10 heads, 4 of them on fair coins, so the probability that the chosen head is on a fair coin is $4/10=2/5$.

20. **E** ($4/9$) $\left(4/5\right)\left(1/5\right)+\left(4/5\right)\left(4/5\right)\left(4/5\right)\left(1/5\right)+\ldots=\frac{\left(4/5\right)\left(1/5\right)}{1-\left(4/5\right)\left(4/5\right)}=4/9$

21. **B** 8 of the 64 small cubes are entirely unpainted, 24 are painted on only 1 side. If the tossed cube is one of the 8 then all 5 showing faces are of course unpainted. If it is one of the 24, then there is a $1/6$ chance that it is unpainted. $8/64+24/64\left(1/6\right)=12/64=3/16$.

22. **C** $1/2\left(4/12\right)+1/2\left(4/6\right)=1/6+1/3=1/2$.

23. **B** $P(A\cup B) = P(A) + P(B) - P(A\cap B) \rightarrow P(A\cap B) = .2$. $P(A|\bar{B}) = P(A\cap \bar{B}) / P(\bar{B})$
$= [P(A) - P(A\cap B)] / [1 - P(B)] = [.3 - .2] / [1 - .5] = .1/.5 = .2$

24. **A** $x^2+bx+c^2=0$ has only real roots when $b^2-4c^2\ge 0 \Leftrightarrow |b|\ge 2|c|$. With the given domains, this can only be true only when $2\le b\le 8$ (also the point $b=-2, c=1$, but the probability of this point is 0). Making a graph of c versus b, our sample space is the rectangle bound by $-2\le b\le 8$ and $1\le c\le 4$, and graph of the values producing real roots is the triangle bound by $(1,2)$, $(4,8)$, and $(1,8)$. The area of this triangle is 9 and the area of the rectangle is 30, so the probability of real roots is $9/30 = 3/10$.

25. **B** $\frac{6}{36}+\frac{28}{36}\left(\frac{6}{36}\right)+\left(\frac{28}{36}\right)^2\left(\frac{6}{36}\right)+\ldots=\frac{6/36}{1-28/36}=\frac{3}{4}$

26. **D** P(5 4’s) = $\left(\frac{1}{6}\right)^5=\frac{1}{6^5}$. P(4 4’s) = $\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^1\binom{5}{4}=\frac{25}{6^5}$. P(4 or 5 4’s) = $\frac{26}{6^5}$.

27. **A**

28. **C** The area under any probability distribution function must be 1. In this case the area is $\frac{c^2}{2}$ (for positive c, otherwise the area is 0). So we solve $\frac{c^2}{2}=1\Rightarrow c=\sqrt{2}$.

29. **C** The possible orderings of his throws are 1, 2, 3; 1, 3, 2; and 3, 1, 2. Each occurs with equal probability, and the 3rd is further than the 1st in $\frac{2}{3}$ of the cases.

30. **D**