Unless otherwise stated, "random" shall indicate a situation in which each case is equally likely to occur.

- 1. Mr. and Mrs. Smith have two boys. If they have a third child, what is the probability it's a girl? A.0 B. $\frac{3}{8}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E.NOTA
- 2. The digits 1, 2, 3, 4, 5, 6 are arranged in random order. What is the probability that the permutation either starts with a 3 or ends with a 4, but not both?

A.
$$\frac{4}{15}$$
 B. $\frac{3}{10}$ C. $\frac{1}{3}$ D. $\frac{9}{10}$ E.NOTA

3. If *n* is a randomly selected natural number less than 2001, what is the probability that *n*! ends with at least 300 zeros?

A.0 B.
$$\frac{501}{2000}$$
 C. $\frac{791}{2000}$ D. $\frac{1697}{2000}$ E.NOTA

4. If two fair six-sided dice are rolled, what is the probability that the sum showing is prime? A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{13}{36}$ D. $\frac{5}{12}$ E.NOTA

5. A 4 digit number is created by randomly selecting 4 digits from the set $\{1, 2, 3, 4, 5, 6\}$ without replacement, what is the probability that this 4 digit number is divisible by 3? A. $\frac{1}{15}$ B. $\frac{1}{5}$ C. $\frac{1}{3}$ D. $\frac{2}{5}$ E. NOTA

6. What is the probability that a random point in a circle of radius 4 is more than 3 units from the center? A $\frac{1}{4}$ B. $\frac{7}{16}$ C. $\frac{1}{2}$ D. $\frac{9}{16}$ E. NOTA

- 7. In a standard 52 card deck, there are 13 ranks, and the deck contains 4 cards of each rank (each of these 4 has a different suit). If I randomly select 2 cards (without replacement), what is the probability that I select a pair of cards with the same rank?
 - A. $\frac{1}{51}$ B. $\frac{1}{17}$ C. $\frac{1}{13}$ D. $\frac{4}{17}$ E. NOTA
- 8. Saahil and Saajan each sit in a random seat in a line of 10 seats (at most one per seat), what is the probability that Saajan is in the seat exactly two seats to Saahil's right?

A.
$$\frac{1}{10}$$
 B. $\frac{1}{9}$ C. $\frac{1}{8}$ D. $\frac{1}{2}$ E. NOTA

9. Three fair coins are tossed, what is the probability that they are not all heads? A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. $\frac{3}{4}$ D. $\frac{7}{8}$ E. NOTA

10. A box contains 3 red balls, 4 white balls, and 5 blue balls. Balls are selected one at a time without replacement until a blue one is chosen. What is the probability that at least 5 balls are selected?

A.
$$\frac{35}{792}$$
 B. $\frac{35}{396}$ C. $\frac{7}{99}$ D. $\frac{7}{44}$ E. NOTA

11. I design 7-digit license plates such that the first 3 characters can be either letters (A-Z) or digits (0-9), while the latter 4 digits can only be digits. What is the probability that a randomly created plate is MAO2006?

A.
$$\frac{1}{36^7}$$
 B. $\frac{1}{36^3(10000)}$ C. $\frac{1}{26^3(10000)}$ D. $\frac{1}{36^3(10)}$ E. NOTA

12. Five boys and five girls are seated randomly around a circular table, what is the probability that no two persons of the same sex are sitting next to each other?

A.
$$\frac{1}{126}$$
 B. $\frac{1}{63}$ C. $\frac{2}{63}$ D. $\frac{2}{35}$ E. NOTA

- 13. Last summer I decided to sail 2,500 miles from Honolulu to Los Angeles. Every day I would wake up and pick a random speed from 0 miles per hour to 50 miles per hour, at which I would travel for either 8 hours or 200 miles, whichever came first (the rest of the day I wouldn't move). What is the expected number of days until I reach LA (rounded up)?
 A.7
 B.13
 C.17
 D.25
 E. NOTA
- 14. A drawer contains 5 red socks and 8 white socks. What is the probability that the first two socks I pull out of the drawer will match?

A.
$$\frac{76}{169}$$
 B. $\frac{19}{39}$ C. $\frac{89}{169}$ D. $\frac{89}{156}$ E. NOTA

15. Two natural numbers are relatively prime if no natural numbers divide both of them (except 1). What is the probability that a random natural number less than 2006 is relatively prime to 2, 3, and 4?

A.
$$\frac{335}{2005}$$
 B. $\frac{502}{2005}$ C. $\frac{585}{2005}$ D. $\frac{669}{2005}$ E. NOTA

- 16. In a game show, there are 3 doors, a random one of which has \$30 behind it, and the others are empty. You guess one door, then the host (who knows where the \$30 is) reveals an empty door that you did not choose, and gives you the option of switching doors. After you decide whether or not to switch, you win whatever is behind the door you ultimately selected. For example, if the money is behind door #1, and you pick door #2, the host will reveal that door #3 has no prize, and will offer you the choice of sticking with door #2 or switching to door #1. What your expected value if you switch doors?

 A.\$10
 B. \$15
 C.\$20
 D.\$30
 E. NOTA
- 17. I have 2 coins. One is two-headed while one has both heads and tails sides. If I randomly choose a coin, then flip it, and it lands heads, what is the probability that it is the two-headed coin?

A.
$$\frac{1}{2}$$
 B. $\frac{2}{3}$ C. $\frac{3}{4}$ D.1 E. NOTA

18. If this test had no correct answers of A, and a student randomly guessed on every question, what is the probability that they get a perfect score (assume that there are no disputes)?

A.0 B.
$$(\frac{1}{4})^{30}$$
 C. $(\frac{1}{5})^{30}$ D. $(\frac{3}{4})^{30}$ E. NOTA

19. A bag contains 8 coins, 3 have heads on both sides, 1 has tails on both sides, and 4 are fair (they have heads on 1 side and tails on the other). If I pull a coin out of the bag and look at one side and it is heads, what is the probability that I selected a fair coin?

A.
$$\frac{1}{4}$$
 B. $\frac{2}{5}$ C. $\frac{1}{2}$ D. $\frac{4}{7}$ E. NOTA

A.

E. NOTA

20. In a certain game, each player has a $\frac{1}{5}$ chance of winning on any one of his or her turns. If a

C. $\frac{3}{5}$

Caltech student and an MIT student play this game, alternating turns until one player wins, with the Caltech student going first, what is the probability that the MIT student wins?

D. 4/5

$$\frac{1}{5}$$
 B. $\frac{1}{2}$

- 21. A cube measuring 4 inches per side is painted on all 6 of its faces, it is then cut into 64 cubes measuring 1 inch per side. A smaller cube is randomly selected and then randomly tossed, what is the probability that all 5 showing faces are unpainted?
 - A. $\frac{1}{8}$ B. $\frac{3}{16}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. NOTA

22. Urn A contains 4 white balls and 8 black balls. Urn B contains 4 white balls and 2 black balls. Neither Urn contains any other balls. First an Urn is randomly chosen, then a ball is randomly chosen from that Urn. What is the probability the ball is white?

A.
$$\frac{1}{3}$$
 B. $\frac{4}{9}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. NOTA

23. Given that P(A) = .3, P(B) = .5, and $P(A \cup B) = .6$, what is $P(A | \overline{B})$? A...1 B..2 C...3 D..7 E. NOTA

24. Given that *b* is a random variable between -2 and 8, and *c* is a random variable between 1 and 4, what is the probability that $x^2 + bx + c^2 = 0$ has only real roots?

A. $\frac{3}{10}$ B. $\frac{2}{5}$ C. $\frac{1}{2}$ D. $\frac{3}{5}$ E. NOTA

25. I'm going to roll 2 fair 6-sided dice until the sum of the top faces is either 7 or 11, what is the probability that I roll a 7 before an 11?

A. $\frac{3}{14}$ B. $\frac{3}{4}$ C. $\frac{9}{10}$ D. $\frac{17}{18}$ E. NOTA

26. If I roll 5 fair 6-sided dice all at the same time, what is the probability that I roll at least 4 6s? A. $\frac{1}{6^5}$ B. $\frac{5}{6^5}$ C. $\frac{6}{6^5}$ D. $\frac{26}{6^5}$ E. NOTA

27. What is the probability of two mutually exclusive events both occurring? A.0 B. $\frac{1}{2}$ C.1 D. Cannot be determined E.NOTA

28. A probability distribution function of a variable, x, is a function such that the area under the graph between x = a and x = b is the probability that $a \le x \le b$. Suppose that the probability distribution function of a random variable, x, is defined as $f(x) = \begin{cases} x & \text{for } x \in [0, c] \\ 0 & \text{otherwise} \end{cases}$. Find c.

A. $\frac{\sqrt{2}}{2}$ B.1 C. $\sqrt{2}$ D. 2 E. NOTA

29. David throws two darts at a dart board, and his 2nd throw is farther from the bulls eye than his 1st. If his skill is constant and he throws a 3rd dart at the board, what is the probability that the 3rd throw is also farther from the center than the 1st?

A. $\frac{1}{3}$ B. $\frac{1}{2}$

D. Cannot be determined E.NOTA

30. If a student randomly guesses one of the 5 answer choices for each question of this test, what is the probability that they get exactly 28 right and 2 wrong (assuming no questions get thrown out)?

 $C.\frac{2}{3}$

A.0 B.
$$(\frac{1}{5})^{28}$$
 C. $(\frac{1}{5})^{28} (\frac{4}{5})^{2}$
D. $(\frac{1}{5})^{28} (\frac{4}{5})^{2} (_{30}C_{2})$ E.NOTA